Midterm Tues. Oct. 27

- Chapters 1-4, excluding ocean-specific sections
  - Composition, Structure, State
  - First and Second Laws of Thermodynamics
  - Transfer Processes
  - Thermodynamics of Water
- In class 80 min (2:00-3:20 pm, NTV 330)
- Closed book
- Constants provided

Quiz Ch. 2

Answer briefly and clearly, with appropriate equations or diagrams.

- What is an exact differential?
- What is the first law of thermodynamics?
- What is reversible work? Give an equation.
- What is entropy? Give an equation.
- Give two examples of “path-dependent” variables.

Lecture Ch. 3a

- Types of transfers
- Radiative transfer and quantum mechanics
  - Kirchoff’s law
  - Blackbody radiation
  - Planck’s radiation law
  - Wien’s displacement law
  - Stefan-Boltzmann law

What are the 3 ways heat can be transferred?

- **Radiation**: transfer by electromagnetic waves.
- **Conduction**: transfer by molecular collisions.
- **Convection**: transfer by circulation of a fluid.

Scalar Transport

- Mass conservation
  - A continuity equation expresses a conservation law by equating a net flux over a surface with a loss or gain of material within the surface.
  - Continuity equations often can be expressed in either integral or differential form.

The conservation of mass is expressed by the continuity equation:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]  

(3.7)

- Transport

\[ \frac{\partial \mathbf{E}}{\partial t} + \nabla \cdot (\mathbf{E} \mathbf{v}) = \frac{1}{\rho} \mathbf{G} \]  

(3.11)

Energy Transport

- Thermodynamic changes with time
  - The time variation of temperature can be written from (2.58) as

\[ \frac{\partial \mathbf{E}}{\partial t} + \nabla \cdot (\mathbf{E} \mathbf{v}) - \frac{\partial \mathbf{G}}{\partial t} = 0 \]  

(3.1)

Using the definition of potential temperature (2.83) for the atmosphere or (2.79) and (2.74) for the oceans, (3.1) becomes

\[ \frac{\partial \mathbf{E}}{\partial t} + \nabla \cdot (\mathbf{E} \mathbf{v}) = \frac{1}{\rho} \mathbf{G} \]  

(3.2)

- Thermodynamic changes with transport

\[ \frac{\partial \mathbf{E}}{\partial t} + \nabla \cdot (\mathbf{E} \mathbf{v}) = \frac{1}{\rho} \mathbf{G} \]  

(3.6)
Sun - our star – the source of most of our energy

For the entire earth, climate can be explained by: 1) the amount of sunlight received and 2) the character of the surface receiving it.

Solar Spectrum

<table>
<thead>
<tr>
<th>Peak</th>
<th>Area Under Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>300K</td>
<td>250 K</td>
</tr>
</tbody>
</table>

Figure 3.1 Black-body irradiance curves for terrestrial temperatures.

Planck’s Radiation Law

- Direct consequence of quantum theory
- The theory of black-body radiation was developed by Planck in 1900. Planck determined a semi-empirical relationship that included the concept that energy is quantized. Planck showed from quantum theory that the black-body radiation $P_{\lambda}$ is given by

$$ P_{\lambda} = \frac{2\pi \lambda^3}{c^2} \left( \exp \left( \frac{hc}{\lambda \sigma T} \right) - 1 \right) $$

where $h$ is Planck’s constant and $k$ is Boltzmann’s constant. Equation (3.19) is known as Planck’s radiation law.

Solar Radiation

- Luminosity of the sun $L_{\odot} \approx 3.9 \times 10^{26}$ W (p. 331)
- Irradiance $I = \frac{L_{\odot}}{A_{\odot}} \approx 1.4 \times 10^{10}$ W/m$^2$ (p. 437)
- Luminosity $L_{\odot} = \epsilon_0 (4\pi r_{\odot}^2) $, where $r_{\odot} = 6.96 \times 10^8$ m
- Estimate blackbody radiation $T_{\odot} \approx (4\pi F_{L_{\odot}} 3800K)$

- Use Wien’s law to evaluate $\lambda_{\text{max}} = 0.5 \mu m$ (visible)
- Similarly, $\lambda_{\text{max}} = 10 \mu m$ (infrared) for $T_{\text{earth}} \approx 300K$

Radiance and Irradiance

\[ F = \iint I \cos \theta \, dA \]

From one direction

- Radiant energy per unit time

\[ I = W \cdot m^{-2} \cdot sr^{-1} \]

Surface area

\[ F \]
Wavelength Dependence

Since the radiant energy is distributed over a spectrum of wavelengths, we define monochromatic radiation, $I_\lambda$, and total radiation, $E_\lambda$ as:

\[ E_\lambda = \int I_\lambda \, d\lambda \quad \text{and} \quad F_\lambda = \int I_\lambda \, d\lambda \]  \hspace{1cm} (3.14)

- **Shortwave**
  - Solar: Wavelengths 0.3-4 µm
- **Longwave**
  - Terrestrial: Wavelengths 4-200 µm

Wien’s Displacement Law

- Inverse dependence of wavelength on temperature

The wavelength of maximum emission for a black body is found by differentiating Planck’s law (3.19) with respect to the wavelength, equating to zero, and solving for the wavelength. This yields

\[ \lambda_{max} = \frac{2hC^2}{kT} \]

This is the location of the peak!

**Radiation Laws - Wien’s Displacement Law**

- Although all known objects emit all forms of electromagnetic radiation, the wavelength of most intense radiation is inversely proportional to the T. (1/T)

**Implications:**
- Sun emits @ ~ 6000 deg Kelvin
- Earth emits @ 288 deg Kelvin,

Which will emit radiation at the longer wavelength?
- Earth

The peak of Solar output is in the visible (light, shorter) part of the electromagnetic spectrum while the Earth emits most of its energy in the infrared (heat, longer) portion of the electromagnetic spectrum.

**Stefan-Boltzmann Law**

- Describes $T^4$ dependence of emission

Integration of (3.19) over all wavelengths gives:

\[ F = \int F_\lambda \, d\lambda = \sigma T^4 \]  \hspace{1cm} (3.20)

where $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ is called the Stefan-Boltzmann constant. Equation (3.20) is referred to as the [Stefan-Boltzmann law](https://en.wikipedia.org/wiki/Stefan%E2%80%93Boltzmann_law), whereby the radiant energy emitted by a black body varies as the fourth power of the absolute temperature. Evaluation of the Stefan-Boltzmann law at $T = 6000$ K (the approximate emission temperature of the Sun) and $T = 300$ K (the approximate emission temperature of the Earth’s surface) shows that $F \approx 6.38 \times 10^{-7}$ W m$^{-2}$ and $F \approx 3.05 \times 10^{-6}$ W m$^{-2}$, a difference of two orders of magnitude.

**Blackbody Radiation**

- Maximum possible emission of radiation

If a body emits the maximum amount of radiation at a particular temperature and wavelength, or equivalently absorbs all of the incident radiation, it is called a black body. For a black body, $A_b = 1$ and $R_b = 0$ for all wavelengths. Black-body radiation is characterized by the following properties:

1. The radiant energy is determined uniquely by the temperature of the emitting body.
2. The radiant energy emitted is the maximum possible at all wavelengths for a given temperature.
3. The radiant energy emitted is isotropic.
Comparison - Earth & Sun Radiation

- Sun - more energy & shorter wavelength
- Earth - lower energy and longer wavelength

Radiation Laws - Black Body Radiation

- Several physical laws describe the properties of electromagnetic radiation that is emitted by a perfect radiator, a so-called black body.
- By definition, at a given temperature, a black body absorbs all radiation incident on it at every wavelength and emits all radiation at every wavelength at the maximum rate possible for a given temperature.
- No radiation is reflected.
- A blackbody is therefore a perfect absorber and a perfect emitter.

Radiation Laws - Black Body Radiation

- The term black body can be misleading because the concept does not refer to color.
- Objects that do not appear black may none the less be blackbodies, perfect radiators.
- Most gases are not blackbodies (see instead Kirchoff’s Law)
- Both the Sun and the Earth closely approximate perfect radiators, so that we can apply blackbody radiation laws to them.
- We’ll discuss 2 laws for blackbody radiation,
  1) Wien's displacement law
  2) Stefan-Boltzmann law.

Radiation Laws - Stefan-Boltzmann law

- Would you expect the same amount of electromagnetic radiation to be emitted by the Earth and Sun?
- No. The total energy radiated by an object is proportional to the fourth power of it's absolute T
  \[ F = k(T^4) = \text{Stefan-Boltzmann law.} \]
  \[ k = \text{Stefan-Boltzmann constant} \ (5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}) \]
- Sun radiates at a much higher temperature than Earth.
- Sun’s energy output/m² = 160,000 that of Earth

Radiant Energy

- Direct
  - Parallel beam
  - One direction
- Diffuse
  - Isotropic
  - All directions

\[ F = \int_{0}^{\infty} \cos \theta \, da \cdot \pi \quad (3.15) \]

Radiative Transfer

- Absorption, Transmission, Reflection

The fraction of the incident radiation that is absorbed \( A \), transmitted \( T \), and reflected \( R \) must add up to unity, so that

\[ A + T + R = 1 \quad (3.15) \]
Energy Balance

Reflected = 25 + 5 = 30
Absorbed = 25 + 45 = 70

Sun’s energy is emitted in the form of electromagnetic radiation (Radiant Energy)
• Radiant energy can interact with matter in 3 ways.
  • Most often its behavior is a combination of two or more of these modes
  • Reflection - there is no change in the matter because of the radiant energy that strikes it and it does not let the energy pass through it (i.e. it is opaque to the radiant energy), then it reflects the energy. Reflection only changes the direction of the beam of radiant energy, not its wavelength or amplitude.
  • Transmission - matter allows radiant energy to pass through it unchanged. Again, there is no change in any of the properties of the radiant energy.
  • Absorption - energy is transferred from the radiant beam to the matter resulting in an increase in molecular energy of the matter.

Reflectivity = Albedo

• Reflected Energy/ Incident Energy
• Higher reflectivity = brighter, shinier surface (snow, ice)
• Lower reflectivity = darker, rougher surface (soil, sand)
• Water - depends on the angle of the sun
• Average albedo for Earth = 30
• Average albedo for moon = 7

Kirchoff’s Law

• Molecules absorb and emit radiation
  • Wavelength determined by quantum mechanics (discrete)

When matter exists as a dilute gas, it absorbs radiation at discrete wavelengths. These spectral lines are characteristic of the gas and correspond to jumps in the quantum energy levels (electronic, vibrational, rotational) of the gas molecule or phonons of either created or absorbed. For matter in the liquid or solid state, molecules are so close to each other that liquids and solids tend to emit and absorb in extended continuous regions of the spectrum rather than in discrete spectral lines and bands.

A molecule that absorbs radiation of a particular wavelength can also emit radiation at the same wavelength. The rate at which emission takes place depends only on the temperature of the matter and the wavelength of the radiation. Kirchoff’s law states that

\[
\frac{F_i}{F_j} = f(\lambda, T)
\]

Answer briefly and clearly, with appropriate equations or diagrams.

Quiz Ch. 3

• What is the approx. wavelength of energy emitted from the Sun?
• Is the Sun’s emissions shortwave or longwave?
• Name one property of blackbody radiation.
• What is the equation for Wien’s law?
• Draw a sketch of Plank’s radiation law.

Curry and Webster, Ch. 3