

## Lecture Ch. 4b

- Hydrostatic equilibrium
  - Special cases
  - Pressure altitude dependence
- More Midterm Review problems
  - Terminology review

Curry and Webster, Ch. 4 (pp. 96-115; skip 4.5, 4.6)  
 Tuesday, Oct. 21: **Homework**, Review (bring questions), and Read Ch. 5  
 Thursday, Oct. 23: **Midterm**  
 Tuesday, Nov. 11 (Veteran's Day): no class (meet to work on ROAST!)  
 Thursday, Nov. 13 Lecture 2:00-3:20 pm on Ch. 7

## More Reminders

- **Virtual Temperature:** The temperature air would have at the given pressure and density **if there were no water vapor in it**
- **Potential Temperature:** The temperature a parcel would have if it were brought **adiabatically and reversibly** to  $p_0$  (usually 1 atm)
- **Virtual Potential Temperature:** The temperature a parcel would have **if there were no water vapor in it (only condensed water)** and if it were brought **adiabatically and reversibly** to  $p_0$  (usually 1 atm)

## Water Vapor Metrics

The *water vapor mixing ratio*,  $w_v$ , is the ratio of the mass of water vapor present to the mass of dry air. It is thus defined, after substituting from the ideal gas law, as

$$w_v = \frac{m_v}{m_d} = \frac{\rho_v}{\rho_d} = \epsilon \frac{e}{p - e} \quad (4.36)$$

where  $\epsilon = M_v/M_d = 0.622$  (Section 1.7). A value of the *saturation mixing ratio*,  $w_s$ , is given by

$$w_s = \epsilon \frac{e_s}{p - e_s} \quad (4.37)$$

Since  $p \gg e$  and  $p \gg e_s$ ,

$$\mathcal{H} \approx \frac{w_v}{w_s} \quad (4.38)$$

is an approximate definition of the relative humidity.

The water vapor mixing ratio can be related to the specific humidity,  $q_v$ , which was originally defined in Section 1.7, as

$$q_v = \frac{m_v}{m_d + m_v} = \epsilon \frac{e}{p - (1 - \epsilon)e} = \frac{w_v}{1 + w_v} \quad (4.39)$$

Since both  $w_v$  and  $q_v$  are always smaller than 0.04,  $q_v \approx w_v$ .

## Ch. 4: Problem 5

Consider moist air at a temperature of 30°C, a pressure of 1,000 hPa, and a relative humidity of 50%. Find the values of the following quantities:

- vapor pressure
- mixing ratio
- specific humidity
- specific heat at constant pressure
- virtual temperature

## Special Cases of Hydrostatic Equilibrium

1.  $\rho = \text{constant}$  (homogeneous)
  - $H = 8 \text{ km} = RT/g = \text{scale height eq. 1.39}$
2. constant lapse rate (implied if hydrostatic, homogeneous, and ideal gas)
  - $-dT/dz = \text{constant} = -g/R = -34 \text{ deg/km}$
3. isothermal  $T = \text{constant}$  (and ideal gas)
  - $p = p_0 \exp(-z/H)$

## Special Cases of Hydrostatic Equilibrium

- Hydrostatic: Force balance on gravity and upward pressure

A pressure gradient force results in a vertical acceleration in the direction of decreasing pressure (upwards). The vertical pressure gradient force is generally in very close balance with the downward force due to gravitational attraction. This is called *hydrostatic balance*, and is written as

$$g = -\frac{1}{\rho} \frac{\partial p}{\partial z} \quad (1.33)$$

where  $g$  is the acceleration due to the Earth's gravity. The hydrostatic balance is applicable to most situations in the atmosphere and ocean, exceptions arising in the presence of large vertical accelerations such as are associated with thunderstorms.

# Homogeneous Atmosphere

Homogeneous  
Constant density  
Constant lapse rate

because air is compressible and density decreases with height in the atmosphere (Figure 1.5), integration of (1.34) for the atmosphere is more complicated than for the ocean. However, useful insights can be derived from examining an idealized homogeneous atmosphere, where density is assumed constant. Consideration of a homogeneous atmosphere with finite surface pressure implies a finite total height for the atmosphere, which is called the *scale height*  $H$ . Assuming that density is constant, we can integrate (1.34) from sea level, where the pressure is  $p_0$ , to a height  $H$ , where the pressure is zero, to obtain

$$\rho_0 = \rho_0 H \quad (1.38)$$

The height of the homogeneous atmosphere (often referred to as the *scale height*) is therefore

$$H = \frac{p_0}{\rho_0 g} = \frac{R_s T_s}{g} \quad (1.39)$$

where  $T_s$  is the surface temperature and  $H$  can be evaluated from the surface temperature and known constants to be approximately 8 km. From the ideal gas law, it is easily inferred that temperature must decrease with height in the homogeneous atmosphere. The lapse rate of the homogeneous atmosphere is obtained by differentiating the ideal gas law with respect to  $z$ , holding density constant

$$\frac{\partial p}{\partial z} = \rho R_s \frac{\partial T}{\partial z}$$

Combining (1.40) with the hydrostatic equation (1.33) leads to the result:

$$\Gamma = -\frac{\partial T}{\partial z} = \frac{g}{R_s} = 34.1 \text{ } ^\circ\text{C km}^{-1}$$

The lapse rate for the homogeneous atmosphere is referred to as the *autoconvective lapse rate* for the following reason: if the lapse rate exceeds the autoconvective value, it is implied that the lower air is less dense than the air above, causing the atmosphere to overturn and the spontaneous initiation of convection. Values of the atmospheric lapse rate as large as the autoconvective value are observed over desert surfaces in summer when the solar heating is high; however, lapse rates in the atmosphere typically do not exceed  $\Gamma = 10^\circ\text{C km}^{-1}$ .

# Isothermal Atmosphere

Further insight is gained by examining the characteristics of yet another idealized atmosphere, called the *isothermal atmosphere*. After substitution of the ideal gas law for density, we can write the hydrostatic equation in the following form:

$$\partial p = -\frac{p g}{R_s T} \partial z \quad (1.42)$$

This equation is easily integrated for a constant temperature from sea level ( $z = 0$ ,  $p = p_0$ ) to some arbitrary height  $z$

$$\int_{p_0}^p \frac{dp}{p} = -\frac{g}{R_s T} \int_0^z dz \quad (1.43)$$

or

$$\ln \frac{p}{p_0} = -\frac{g z}{R_s T} \quad (1.44a)$$

Taking antilogs and using  $H = RT/g$ , we have

$$p = p_0 \exp(-z/H)$$

This pressure decreases exponentially with height in an isothermal atmosphere, and there is no definite upper boundary to this atmosphere. Note that when  $z = H$ , the pressure is  $1/e$  of its surface value. The isothermal atmosphere resembles the real atmosphere more closely than does the homogeneous atmosphere; however, (1.44a) is not applicable to the real atmosphere except when applied over a shallow layer above the ground.

# Hydrostatic Equilibrium Example

Consider a planet with an atmosphere in hydrostatic equilibrium. Assume that the atmosphere is an ideal gas. Also assume that the temperature is a maximum at the surface of the planet, and, as height increases, the temperature in the atmosphere decreases linearly (in other words, temperature decreases with height at a constant rate). Derive a formula for atmospheric density as a function of height in this atmosphere.

# Pressure Altitude Calculator

Let's compare the hydrostatic equation to the atmosphere

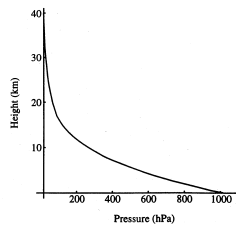
$$p = p_0 \left( \frac{T}{T_0} \right)^{g/R_s \Gamma}, T = T_0 - \Gamma z \rightarrow p = p_0 \left( \frac{T_0 - \Gamma z}{T_0} \right)^{g/R_s \Gamma}$$

Hydrostatic Equation

Surface Temperature	298 K
Lapse Rate	0.0065 K/m
Surface Pressure	1013 mbar or hPa
Altitude	10000 m
Rd	287 J/K/kg
Calculated Pressure	272.62518 mbar or hPa

<http://www.csgnetwork.com/pressurealtcalc.html>

# Pressure-Altitude Dependence



# Latitudinal and Seasonal Variability of Pressure-Altitude

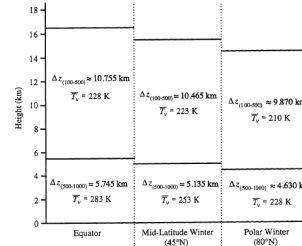


Figure 1.12 Representation of the thicknesses of the 1000–500 hPa and 500–100 hPa layers and their variation with latitude. The thickness of the layer between two isobaric surfaces is determined by the mean virtual temperature in the layer, according to (1.45), resulting in layers of decreasing thickness from equator to pole.