Lecture Ch. 4b

- Homework Problem Ch. 4 Prob. 5
- Hydrostatic equilibrium
  - Special cases
  - Pressure altitude dependence
- More Midterm Review problems
  - Terminology review

Curry and Webster, Ch. 4 (pp. 96-115; skip 4.5, 4.6)
For Tuesday: Read Ch. 5

More Reminders

- **Virtual Temperature:** The temperature air would have at the given pressure and density if there were no water vapor in it
- **Potential Temperature:** The temperature a parcel would have if it were brought adiabatically and reversibly to p_0 (usually 1 atm)
- **Virtual Potential Temperature:** The temperature a parcel would have if there were no water vapor in it (only condensed water) and if it were brought adiabatically and reversibly to p_0 (usually 1 atm)

Ch. 4: Problem 5

Consider moist air at a temperature of 30°C, a pressure of 1,000 hPa, and a relative humidity of 50%. Find the values of the following quantities:

a) vapor pressure
b) mixing ratio
c) specific humidity
d) specific heat at constant pressure
e) virtual temperature

Hydrostatic Equilibrium Example

Consider a planet with an atmosphere in hydrostatic equilibrium. Assume that the atmosphere is an ideal gas. Also assume that the temperature is a maximum at the surface of the planet, and, as height increases, the temperature in the atmosphere decreases linearly (in other words, temperature decreases with height at a constant rate). Derive a formula for atmospheric density as a function of height in this atmosphere.

Special Cases of Hydrostatic Equilibrium

- 1. rho=constant (homogeneous)
  - H=8 km =RT/g=scale height eq. 1.39
  - DT/dz=-g/R=-34/deg/km
- 2. constant lapse rate
  - -dT/dz=constant
- 3. isothermal T=constant
  - p=p_0*exp(-z/H)

Pressure Altitude Calculator

Let's compare the hydrostatic equation to the atmosphere

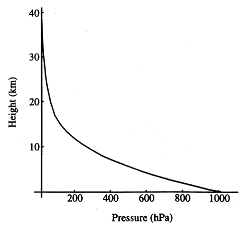
$$p = p_0\left(\frac{T}{T_0}\right)^{g/R_d\Gamma}, T = T_0 - \Gamma z \rightarrow p = p_0\left(\frac{T_0 - \Gamma z}{T_0}\right)^{g/R_d\Gamma}$$

Hydrostatic Equation

Surface Temperature	294	K
Lapse Rate	0.0065	K/m
Surface Pressure	1013	mbar or hPa
Altitude	10000	m
Rd	287	J/K/kg
Calculated Pressure	272.62518	mbar or hPa

http://www.csgnetwork.com/pressurealtcalc.html

Pressure-Altitude Dependence



Latitudinal and Seasonal Variability of Pressure-Altitude

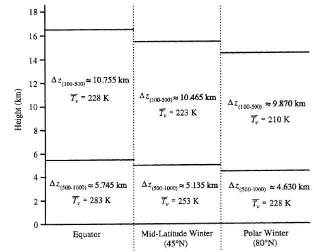


Figure 1.12 Representation of the thicknesses of the 1000–500 hPa and 500–100 hPa layers and their variation with latitude. The thickness of the layer between two isobaric surfaces is determined by the mean virtual temperature in the layer, according to (1.45), resulting in layers of decreasing thickness from equator to pole.

Radiation Balance Example

It has been estimated from satellite observations that variations in solar radiance during the last 20 years amounted to ≤0.2 W m^{-2}, or less than 0.1% of the incoming shortwave radiation. Calculate the approximate change in the temperature at the Earth's surface for a 0.1% decrease in solar luminosity for a simplified climate model. State all assumptions, simplifications, and equations used. Values of constants that you may need are Earth's albedo 0.31, solar luminosity 3.92×10^{26} W, Earth-sun distance 1.50×10^{11} m, Stefan-Boltzmann constant 5.67×10^{-8} W m^{-2} K^{-4}.

Definition Example

Define the following terms, briefly and clearly, in light of their use in the kinetic theory of gases and the first and second laws of thermodynamics:

a) an ideal gas
b) temperature
c) entropy
d) exact differential
e) enthalpy

Terminology Review

- Synoptic
  - large phenomena, hundreds of kilometers in length
- Isentropic
  - Adiabatic+reversible
- For adiabatic, ideal:
  - p determines T and vice versa
- Potential temperature
  - temperature that air would have if raised/lowered to a reference pressure.

Hydrostatic Equilibrium Example

Consider a planet with an atmosphere in hydrostatic equilibrium. Assume that the atmosphere is an ideal gas. Also assume that the temperature is a maximum at the surface of the planet, and, as height increases, the temperature in the atmosphere decreases linearly (in other words, temperature decreases with height at a constant rate). Derive a formula for atmospheric density as a function of height in this atmosphere.

From the hydrostatic equation for an ideal gas (Eqn. 1.42)

$$\partial p = -\frac{pg}{R_d T}\partial z$$

and a constant lapse rate $\Gamma = -\frac{dT}{dz}$ we get

$$dp = -\frac{pg}{R_d T}dz\left(\frac{-dT/dz}{\Gamma}\right) = -\frac{pg}{\Gamma R_d}\frac{dT}{T}$$

$$\frac{dp}{p} = \left(\frac{g}{\Gamma R_d}\right)\frac{dT}{T}$$

$$\int\frac{dp}{p} = \int\left(\frac{g}{\Gamma R_d}\right)\frac{dT}{T}$$

$$\ln\frac{p}{p_0} = \left(\frac{g}{\Gamma R_d}\right)\ln\frac{T}{T_0}$$

$$p = p_0\left(\frac{T}{T_0}\right)^{\left(g/R_d\Gamma\right)}$$

which is Eqn 1.48. Then dividing both sides by RT and noting that for an ideal gas

$\rho = \frac{p}{RT}$, we get

$$\frac{p}{RT} = \rho = \frac{p_0}{RT}\left(\frac{T}{T_0}\right)^{g/R_d\Gamma} = \frac{p_0}{R(T_0-\Gamma z)}\left(\frac{T_0-\Gamma z}{T_0}\right)^{g/R_d\Gamma}$$

Definition Example

Define the following terms, briefly and clearly, in light of their use in the kinetic theory of gases and the first and second laws of thermodynamics:

a) an ideal gas: vapor whose molecules have collisions with perfect elasticity, typical at low pressures (for air ≤1 atm) and high temperatures (typically ≥300K); vapor that satisfies pv=RT and has the properties that $dh=c_p dT$, $du=c_v dT$, $c_p-c_v=R$ (p. 44).

b) temperature: the intensive property describing the internal energy of a gas, which for an ideal gas depends only on the average speed of the molecules.

c) entropy: a state property whose differential describes the amount of energy that is not available for doing work for a reversible process (in which maximum work is done) and satisfies the criteria of an exact differential; it can be evaluated from Eqn. 2.25a and 2.26b:

$$d\eta = \left(\frac{dQ}{T}\right)_{rev} = c_p d(\ln T) - Rd(\ln p).$$

d) exact differential: a function ξ for which dξ has the properties (1) for any closed path $\oint d\xi = 0$, and (2) for ξ(x,y) where x and y are independent, then

$$d\xi = \left(\frac{\partial \xi}{\partial x}\right)_y dx + \left(\frac{\partial \xi}{\partial y}\right)_x dy \equiv Mdx + Ndy \Rightarrow \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y.$$

e) enthalpy: a state property whose differential describes the change in heat for a constant-pressure process and satisfies the criteria of an exact differential; it is defined as H=U+pV (Eqn. 2.12).

Ch. 4: Problem 5

Consider moist air at a temperature of 30°C, a pressure of 1,000 hPa, and a relative humidity of 50%. Find the values of the following quantities:

a) vapor pressure
b) mixing ratio
c) specific humidity
d) specific heat at constant pressure
e) virtual temperature

The saturation vapor pressure at a temperature of 30°C is 42.4 hPa. The gas constant for dry air is 287 J K^{-1} kg^{-1}. The gas constant for water vapor is 461 J K^{-1} kg^{-1}.

a) vapor pressure: $e=H*e_s = 0.50*42.4 = 21.2$ hPa [Eqn. 4.34a]
b) mixing ratio: $w_v=m_v/m_d=(M_v/M_d)*(e/(p-e))=0.0135$ [Eqn. 4.36]
c) specific humidity: $q_v=m_v/(m_v+m_d)=w_v/(w_v+1)=0.0133$ [Eqn. 1.20]
d) specific heat at constant pressure: $c_p \sim (7R/2)(1+0.87q_v)=1016$ J/K/kg [Eqn. 2.65]
e) virtual temperature: $T_v=(1+0.608q_v)*T=305.6$ K [Eqn. 1.25]

Radiation Balance Example

It has been estimated from satellite observations that variations in solar radiance during the last 20 years amounted to ≤0.2 W m^{-2}, or less than 0.1% of the incoming shortwave radiation. Calculate the approximate change in the temperature at the Earth's surface for a 0.1% decrease in solar luminosity for a simplified climate model. State all assumptions, simplifications, and equations used. Values of constants that you may need are Earth's albedo 0.31, solar luminosity 3.92×10^{26} W, Earth-sun distance 1.50×10^{11} m, Stefan-Boltzmann constant 5.67×10^{-8} W m^{-2} K^{-4}.

Assume that: (1) the earth behaves as a blackbody, (2) atmosphere is transparent to non-reflected portion of the solar beam; (3) atmosphere in radiative equilibrium with surface; (4) atmosphere absorbs all the infrared emission. Then, at equilibrium, the incoming shortwave flux and outgoing longwave flux are equal (i.e. there is no accumulation) so for the normal solar luminosity we can write:

$F_L = \sigma T_{surf}^4$ (assumption 1; Eqn. 3.20)
$F_S = F_L$(assumption 2-4)
$0.25*S_0(1-\alpha_p) = \sigma T_{surf}^4$ (Eqn. 12.1b)
$T_{surf} = [0.25*S_0(1-\alpha_p)/\sigma]^{0.25}$
where $S_0 = L_0/(4\pi d^2) = 1.3938 \times 10^3$ W m^{-2} (p. 332), $\alpha_p=0.31$, $\sigma=5.67 \times 10^{-8}$ W m^{-2} K^{-4}
T_{surf} = 254.84K
Reducing L to 99.9% of its value, we get $S_0' = 0.999*1.3938 \times 10^3 = 1.3924 \times 10^3$ W m^{-2}
T_{surf} = 254.78K
The resulting cooling of 0.06K associated with a 0.1% reduction in solar radiation is negligible for this simplified climate model.

Clausius Clapeyron Example

The saturation vapor pressure at a temperature of 30°C is 42.4 hPa. The gas constant for dry air is 287 J K^{-1} kg^{-1}. The gas constant for water vapor is 461 J K^{-1} kg^{-1}.

In addition to the constants given above, here is one more: the saturation vapor pressure at a temperature of 40°C is 73.8 hPa. Assuming that the latent heat of vaporization is constant, use this information to calculate the numerical value for this latent heat.

The Clausius Clapeyron equation can be integrated if L is assumed constant, and the result is Eqn. 4.23. Using 30°C=303K and 40°C=313K, and knowing saturation vapor pressure values for each, the only unknown is L. Solving Eqn. 4.23,

$$e_2 = e_1 \exp\left[-\frac{L_{lv}}{R_v}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right]$$

$$L_{lv} = -R_v\left(\frac{T_1 T_2}{T_1 - T_2}\right)\left(\ln\frac{e_2}{e_1}\right) = -461\left(\frac{303 \times 313}{303 - 313}\right)\left(\ln\frac{73.8}{42.4}\right) = 2.4 \times 10^6$$

$L=2.4 \times 10^6$ J/kg.

Degrees of Freedom Example

Name the five main components of the atmosphere. (a) If all components are in the gas phase, how many degrees of freedom are there in the system? (b) If water condenses or freezes, does that number increase or decrease? (c) If new components are added by pollution, how does that change (i) the number of possible phases and (ii) the degrees of freedom of the atmosphere?

The five main components of the atmosphere are nitrogen (N_2), oxygen (O_2), carbon dioxide (CO_2), argon (Ar), and water (H_2O).

(a) For this system, we can use the Gibbs phase rule (Eqn. 4.2) with $\chi=5$, $\varphi=1$, $f=\chi-\varphi+2=6$.

(b) Condenses $\varphi=2$, f=5 [decrease]; freezes $\varphi=2$, f=5 [decrease] (both: $\varphi=3$, f=4 [decrease]).

(c) (i) number of phases that can exist at atmospheric pressure may increase with additional components, since multiple liquid and solid phases may form; (ii) degrees of freedom increase with the number of components and will decrease with the number of phases.

Quiz

Answer briefly and clearly, with appropriate equations or diagrams.

- What makes the chemistry of water unique?
- What is a degree of freedom?
- What is the relationship between two pressures at mechanical equilibrium?
- What is a supersaturated solution?
- What conditions are required for the Gibbs phase rule? (Be specific; name three.)

Curry and Webster, Ch. 4