History of the Standard Atmosphere

• With a little digging, you can discover that the Standard Atmosphere can be traced back to [1219]. The constant lapse rate of 6.5° per km in the troposphere was suggested by Prof. Tousant, on the grounds that...what is needed is...merely a law that can be conveniently applied and which is sufficiently in accordance with the means adhered to. By this method, corrections due to temperature will be as small as possible in calculations of airplane performance, and will be easy to calculate.

• The above quotation is from the paper by Gregg (1912). The early motivations for this simplified model were evidently the calibration of aneroid altimeters for aircraft, and the construction of firing tables for long-range artillery, where air resistance is important.

• Unfortunately, it is precisely the inaccurate region below 1000 m that is most important for navigation near the horizon. However, the standard lapse rate, which Gregg calls "arbitrary," is now embodied in so many altimeters that it cannot be altered; all revisions of the Standard Atmosphere have preserved it.

• Therefore, the Standard Atmosphere is really inappropriate for astronomic refraction calculations. A more realistic model would include the diurnal changes in the boundary layer; but these are still poorly understood that no satisfactory basis seems to exist for realistic refraction tables near the horizon.

http://terma.wisc.edu/21/explanation/standard_atmosphere.html

International Standard Atmosphere

• The ISA model divides the atmosphere into layers with linear temperature distributions.[2] The other values are crangel from base, physical constants and relationships. Thus the standard consists of a table of values at various altitudes, plus some formulas by which these values were derived. For example, at sea level the standard uses a pressure of 1.013 bar and a temperature of 15°C, and an initial lapse rate of 6.5 °C/km. Above 12 km the tabulated temperature is uniformly constant. The tabulation continues to 100 km where the pressure has fallen to 0.075 bar and the temperature to -56.5 °C.[3][4]

<table>
<thead>
<tr>
<th>Layer in the ISA</th>
<th>Base</th>
<th>Basic Geopotential Height (in km)</th>
<th>Basic Geopotential Altitude (in m)</th>
<th>Lapse Rate in Temperature (°C/km)</th>
<th>Lapse Rate in Pressure (hPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
<td>1.013</td>
</tr>
<tr>
<td>1</td>
<td>11.0</td>
<td>11.0</td>
<td>0.0</td>
<td>6.5</td>
<td>1.103</td>
</tr>
<tr>
<td>2</td>
<td>20.0</td>
<td>20.0</td>
<td>0.0</td>
<td>6.5</td>
<td>1.639</td>
</tr>
<tr>
<td>3</td>
<td>32.0</td>
<td>32.0</td>
<td>2.0</td>
<td>11.3</td>
<td>1.682</td>
</tr>
<tr>
<td>4</td>
<td>47.0</td>
<td>47.0</td>
<td>6.0</td>
<td>11.3</td>
<td>1.100</td>
</tr>
<tr>
<td>5</td>
<td>61.0</td>
<td>61.0</td>
<td>6.0</td>
<td>28.4</td>
<td>0.884</td>
</tr>
<tr>
<td>6</td>
<td>71.0</td>
<td>71.0</td>
<td>2.3</td>
<td>38.5</td>
<td>0.984</td>
</tr>
<tr>
<td>7</td>
<td>84.0</td>
<td>84.0</td>
<td>2.3</td>
<td>48.6</td>
<td>1.012</td>
</tr>
</tbody>
</table>

Notes:

Geopotential Height

Geopotential height is a vertical coordinate referred to Earth’s mean sea level — an adjustment to geopotential height by subtracting the variation of gravity with latitude and elevation. This is taken as the “gravity adjusted height.” One usually speaks of the geopotential height of a certain pressure level, which would correspond to the geopotential height of the given pressure.

At an elevation of h, the geopotential is defined as

\[ \Phi = \int_0^h g(z) \, dz, \]

where \( g(z) \) is the acceleration due to gravity, \( z \) is latitude, and \( h \) is the geopotential elevation. Thus, \( \Phi \) is the potential energy per unit mass at that level. The geopotential height is

\[ Z_g = \frac{\Phi}{g_0}, \]

where \( g_0 \) is the standard gravity at mean sea level.

Geopotential height is the geopotential height rather than geodetic height, because doing so is more convenient in some geological calculations more convenient. For example, the presence of geopotential which would correspond to in some places of the same pressure level. Using these forces estimates net vertical forces and no density which is very difficult to use in the equations.

ICAO Standard Atmosphere

The International Civil Aviation Organization (ICAO) Standard Atmosphere gives the average values for meteorological elements at 40°N from mean sea level (MSL) to 80 km (262,500 ft). The ICAO Standard Atmosphere does not contain water vapor.

Some of the values defined by ICAO are:

ICAO Standard Atmosphere

<table>
<thead>
<tr>
<th>Height km</th>
<th>Temperature °C</th>
<th>Pressure hPa</th>
<th>Lapse Rate °C/km</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15.0</td>
<td>1013.25</td>
<td>1.98 (Troposphere)</td>
</tr>
<tr>
<td>1 km</td>
<td>15.5</td>
<td>1013.25</td>
<td>1.00 (Stratosphere)</td>
</tr>
<tr>
<td>2 km</td>
<td>15.8</td>
<td>1013.25</td>
<td>0.00 (Stratosphere)</td>
</tr>
<tr>
<td>3 km</td>
<td>16.0</td>
<td>1013.25</td>
<td>1.00 (Stratosphere)</td>
</tr>
</tbody>
</table>

In this is a standard, you will never encounter these conditions outside of a laboratory, but many Aviation standards and flying rules are based on this, being useful for one major. The standard is very useful in Meteorology for comparing actual values to.

Lecture Ch. 2a

• Energy and its properties
  - State functions or exact differentials
  - Internal energy vs. enthalpy

• First law of thermodynamics

• Heat/Work cycles
  - Energy vs. heatwork?
  - Adiabatic processes
  - Reversible "P-V" work

• Homework problem Ch. 2, Prob. 2

Curry and Webster, Ch. 2 pp. 35-47
Van Ness, Ch. 2

Internal Energy vs. Enthalpy

It is convenient to define a new function called the enthalpy, \( h \), by

\[ h = u + pv \]

(2.12)

• Difference b/w \( U \) and \( H \)
  - \( U \) depends on \( u = \left( \frac{\partial u}{\partial T} \right)_p + \left( \frac{\partial u}{\partial p} \right)_T dp \)
  - \( H \) depends on \( h = \left( \frac{\partial h}{\partial T} \right)_p + \left( \frac{\partial h}{\partial p} \right)_T dp \)

• Specific heats [a.k.a. heat capacity]
  - \( c_p \) is constant \( v \)
    \[ c_p = \frac{\partial h}{\partial T} \] (2.15a)
  - \( c_v \) is constant \( p \)
    \[ c_v = \frac{\partial u}{\partial T} \] (2.15b)
Heat Capacity

\[ ds = c_v \, dT \]

\[ dh = c_p \, dT \]

(2.16)

How does \( c_p \) differ from \( c_v \) qualitatively? In a constant pressure process, some of the added heat must be expended in doing work on the surroundings, while in a constant-volume process, all of the heat is devoted to raising the temperature of the substance. Therefore it takes more heat per unit temperature rise at constant pressure than at constant volume, and \( c_p > c_v \). The difference between \( c_p \) and \( c_v \) can be evaluated from

\[ c_p - c_v = \left( \frac{\partial \ln P}{\partial T} \right)_V \]

For an ideal gas

- Simplify to
- \( \text{The equation of state is } pV = RT \).
- \( \text{The internal energy is a function of its temperature alone } \Rightarrow ds = c_v \, dT \).
- \( \text{The specific heats are related by } c_p - c_v = R. \)
- \( \text{[Types of processes]} \)
  - Constant pressure
  - Constant volume

Lord Kelvin
(a.k.a. William Thomson)

James P. Joule

- The First Law of Thermodynamics
  \[ dU = dQ + dW \]

- Consequences
  - Uniqueness of work values \( W_{rev} = - \int pdV \)
  - Reversible
  - Adiabatic
  - Conservation of energy \( Q = 0 \Rightarrow W = 0 \)
  - See also 2nd law!
  - Impossibility of perpetual motion machine \( Q = 0 \Rightarrow W = 0 \)
  - State function
  - Relativity
  - \( \Delta E = mc^2 \)
  - Proof follows...

Other Kinds of Energy

- In addition to changes in internal energy, a system may change:
  - Potential energy for height change \( \Delta z \)
  - Kinetic energy for velocity change \( \Delta v \)
  - Nuclear energy for mass change \( \Delta m \)

\[ \Delta E = \Delta U(p, V, T) + mg \Delta z + \frac{1}{2} m \Delta v^2 - c^2 \Delta m = Q + W \]

if \( \Delta E = \Delta U(p, V, T) \), then \( \Delta U(p, V, T) = Q + W \)

Van Ness, p. 13

Work

- Expansion work \( W = -pdV \) or \( w = -pdv \)
  - Lifting/rising
  - Mixing
  - Convergence
- Other kinds of work?
  - Electrochemical (e.g. batteries)

![Diagram of work types](image)

Figure 2.1: Rising motion occurs in the atmosphere due to (a) orographic lifting, (b) frontal lifting, (c) low-level convergence, (d) boundary layer rising of warm air, and (e) mechanical mixing. Expansion work is done by air parcels when they rise.
Cycles

- Work and heat are path-dependent transfers
  - W work
  - Q heat
- State functions are unique "states"
  - U internal energy
  - H enthalpy
  - η (also S) entropy
  - A Helmholtz free energy

Exact Differentials

- State functions are exact differentials

An exact differential \( df \) has the following properties:
1. The integral of \( df \) along a closed path is equal to zero (\( df \rightarrow 0 \)).
2. For \( f(x, y) \), we have \( df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy \) where \( x \) and \( y \) are independent variables of the system and the subscripts \( x \) and \( y \) on the partial derivatives indicate which variable is held constant in the differentials.

Figure 2.2 (a) The amount of work done in the expansion from \( V_i \) to \( V_f \) is equal to the area under the curve. In (b), the system is compressed back to \( V_i \) via a different process. Even though the system has returned to its initial state, net work has been done, as indicated by the shaded area between the two curves.

Figure 2.6 Carnot heat engine. Heat \( q_1 \) is brought from the hot reservoir to the engine. The engine does work \( w \) and rejects heat \( q_2 \) into the cold reservoir.

Heat/Work Cycles

- The efficiency with which work is accomplished in a reversible cyclic process depends only on the temperature of the reservoirs to which heat is transferred

THE CARNOT CYCLE

STEP 1: Expand isothermally and reversibly at \( T_1 \)
STEP 2: Compress isothermally and reversibly at \( T_2 \)
STEP 3: Expand adiabatically and reversibly at a T
STEP 4: Compress adiabatically and reversibly

Efficiency: \( \eta = 1 - \frac{T_2}{T_1} \)

Figure 2.3 Isothermal expansion compared with a reversible adiabatic expansion. For a given drop in pressure, \( \Delta V_{\text{isoth}} > \Delta V_{\text{adiab}} \) since during the adiabatic expansion, the temperature also decreases.
P-V diagrams of work

• Work is determined by pathway

Other Work Cycles

Nikolaus Otto developed the Otto cycle in 1876.

Rudolf Diesel developed the Diesel cycle in 1892.

The Diesel Cycle works by compressing air and then adding fuel directly to the piston. The compressed air then combusts the mixture.

The Otto Cycle works by compressing a mixture of air and fuel in a piston and then igniting the mixture with a spark.

Efficiency:

\[ \eta = 1 - \frac{T_2 - T_1}{T_1 - T_0} \]

4 Steps of Carnot “Engine”

1: Add Heat (isothermally)
2: Adiabatic
3: Lose Heat (isothermally)
4: Adiabatic

Hurricane as Carnot Cycle

Ideal Gases

2.4 Applications of the First Law to Ideal Gases

We now apply the first law of thermodynamics to ideal gases, which is useful in the interpretation of thermodynamic processes in the atmosphere. The thermodynamic characteristics of an ideal gas have been shown to be:

1. The equation of state is \( pV = nRT \).
2. The internal energy is a function of its temperature alone: \( (dU = c_v dT) \).
3. The specific heats are related by \( c_p - c_v = R \).

\[ c_p = \frac{\partial h}{\partial T}\bigg|_p = \frac{dh}{dT} = \frac{\partial h}{\partial T}\bigg|_V \]

Reversible-Adiabatic-Work

Reversible, Adiabatic

\[ T_1^a \left[ \frac{p}{p_1} \right]^\gamma = T_2 \]

Frictionless

Low P, Low T

First Law

\[ \Delta u = Q + W \]

Ideal Gas

Adiabatic

mass is conserved

\[ P_v \frac{V}{T} = \frac{P_0 V_0}{T_0} \]

Internal Energy

\[ \Delta u = c_v dT \]

Reversible, Adiabatic
Reversible Processes

- Always at or infinitesimally close to equilibrium
- Infinitesimally small steps
- Infinite number of steps
- Each step can be reversed with infinitesimal force

\[ W_{\text{rev}} = -p \, dv \]

Frictionless mass is conserved

Figure 2.4. Comparison of a reversible and an irreversible process in the atmosphere. In the reversible case, the system passes through an infinite number of infinitesimally small steps each of which can be reversed with infinitesimal force. In the irreversible process, the system does not remain at equilibrium, and the total work done is greater.

Homework Ch. 2 Problem 2

2. A unit mass of dry air undergoes a Carnot cycle consisting of the following steps:

   a) adiabatic compression from 60 kPa and 30°C to temperature of 20°C.
   b) isothermal expansion to a pressure of 70 kPa.
   c) adiabatic expansion to a temperature of 30°C.
   d) isothermal compression to the original pressure of 60 kPa.

   Calculate the work done by the air in kJ/kg.

"The intercooler removes heat from the turbocharged air intake manifold, thereby increasing the density, as the air intake increases in efficiency, the gas temperature in the combustion chamber falls, suppressing the occurrence of knocking, and increasing engine output."