11/7 Conserved quantities
↳ analogous to θ
↳ conserved during ascent of the airmass
↳ calculate initial value at surface, don't need to calculate again aloft; or, can calculate other quantities (like T) aloft based on the conserved quantity

Equivalent ~~Virtual~~ potential temperature:

Reminder: • virtual potential temp – account for presence of water vapor
• equivalent: account for latent heating by condensation of water vapor
• potential temp: account for adiabatic expansion

Assume saturation is maintained throughout parcel ascent; so phase equilibrium applies. Adiabatic process:

$$c_p \, d\log T - R \, d\log p + d\left(\frac{L w_s}{T}\right) = 0 = dq \quad (*)$$

$$\theta = T\left(\frac{p_0}{p}\right)^{R/c_p} \to \log\theta = \log T - \frac{R}{c_p}\log p$$
$$\to d\log\theta = d\log T - \frac{R}{c_p} d\log p$$

rewriting (*) in terms of θ,

$$c_p \, d\log\theta = -d\left(\frac{L w_s}{T}\right)$$

"Equivalent" potential temperature is the potential temperature a parcel <u>would</u> have if all water vapor were condensed out by adiabatic lifting:

$$\int_{\log\theta}^{\log\theta_e} c_p \, d\log\theta' = -\int_{w_s}^{0} d\left(\frac{\mathcal{L} w_s'}{T}\right)$$

$$\Leftrightarrow \quad c_p \log\frac{\theta_e}{\theta} = \frac{\mathcal{L} w_s}{T}$$

$$\Rightarrow \quad \theta_e = \theta \exp\left(\frac{\mathcal{L} w_s}{c_p T}\right)$$

$$\frac{\mathcal{L} w_s}{c_p T} > 0 \Rightarrow \theta_e > \theta$$

because θ_e always receives a contribution from latent heating

Now let's show that θ_e is conserved (i.e., $\frac{d\theta_e}{dz} = 0$)

$$\cancel{\frac{d\theta_e}{dz}} \quad d \log \theta_e = d \log \theta + d \frac{L w_s}{c_p T}$$

$$\Rightarrow \qquad = d \log T - \frac{R}{c_p} d \log p + d\left(\frac{L w_s}{c_p T}\right)$$

$$\Rightarrow c_p \, d \log \theta_e = c_p \, d \log T - R \, d \log p + d\left(\frac{L w_s}{T}\right)$$

$$= 0$$

$$\Rightarrow c_p \frac{d \log \theta_e}{dz} = 0 \quad \Rightarrow \quad \frac{d\theta_e}{dz} = 0$$

... to the extent that the entropy equation (*) holds.

Moist static energy:

useful if w_ℓ can no longer be neglected (such as very wet tropical clouds)

$$c_p\, dT \rightarrow (c_p + w_\ell c_\ell)\, dT$$

$$v\, dp = g\, dz \rightarrow g(1+w_\ell)\, dz$$

⇒ entropy equation →

$$\underbrace{(c_p + w_\ell c_\ell)\, dT + d(L w_v) + g(1+w_\ell)\, dz}_{\equiv dh} = 0$$

$$\rightarrow \boxed{h = (c_p + w_\ell c_\ell) T + L w_v + g(1+w_\ell) z}$$

given an initial air parcel, $z=0$, w_v, w_ℓ, T, can calculate how high it will rise by condensing all water vapor; example: tropical convection

11/5 Parcel modeling

↳ parcel model assumptions

↳ coupled differential eq.s with dynamical variables T, T', w_l, w_v, p (or z)

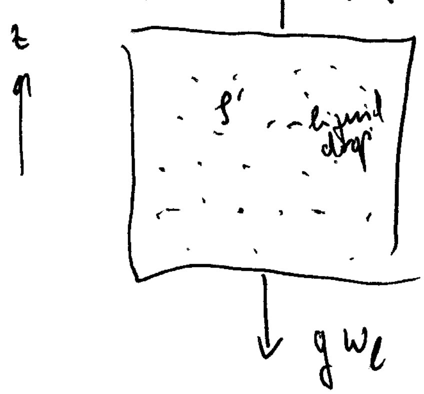
- parcel does not mix with environment
- parcel does not disturb the environment (drag forces ~~etc~~, turbulence, ~~etc~~ sinking, etc.)
- parcel and ambient pressures are equal
- parcel is adiabatic

How valid are these assumptions? What if the volumes involved are very large?

Discussion: Why model (instead of using eq.'s from earlier)?

Two equations govern the parcel:

① Buoyancy $\uparrow g\left(\frac{\rho^E - \rho'}{\rho'}\right)$



$$\frac{dW}{dt} = \frac{d^2z}{dt^2} = g\left(\frac{\rho^E - \rho'}{\rho'} - w_l\right)$$

$$= g\left(\frac{T' - T}{T} - w_l\right)$$

NB: this model is ~~not~~ "hydrostatic"!
Discuss: what would make it non-hydrostatic?

② Energy conservation

$$\left(c_p + L\frac{dw_s}{dT}\right)dT = -g\,dz = -gW\,dt$$

These equations can be integrated numerically

$$g\,\frac{\rho - \rho'}{\rho'}$$

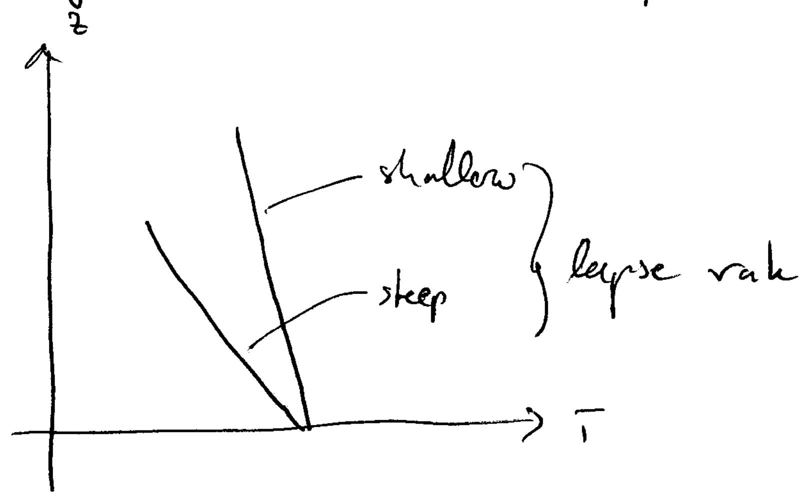
$$pv = RT \rightarrow \frac{p}{\rho} = RT$$

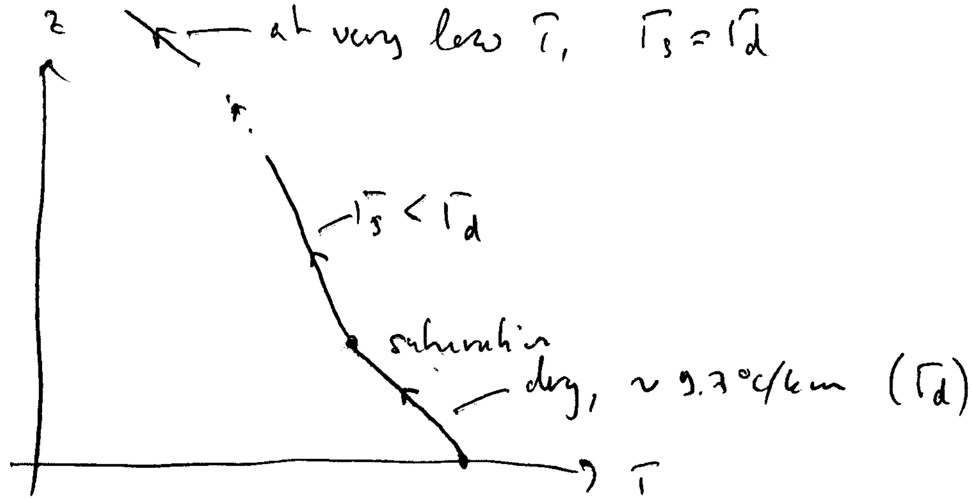
$$\rightarrow \rho = \frac{p}{RT}$$

$$= g\,\frac{\frac{p}{RT} - \frac{p}{RT'}}{\frac{p}{RT'}} = g\,\frac{\frac{1}{T} - \frac{1}{T'}}{\frac{1}{T'}}$$

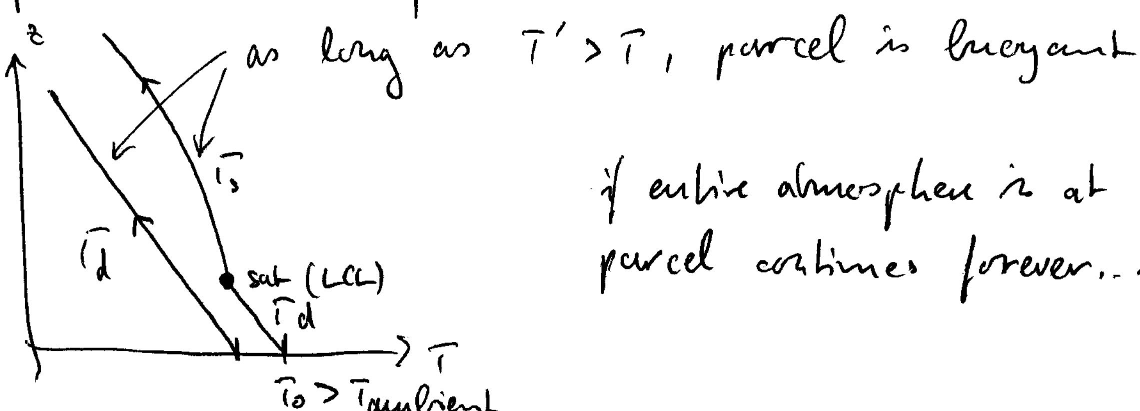
$$= g\,\frac{\frac{TT'}{T} - \frac{TT'}{T'}}{\frac{TT'}{T'}} = g\,\frac{T' - T}{T}$$

Modeling a parcel's ascent: temperature evolution

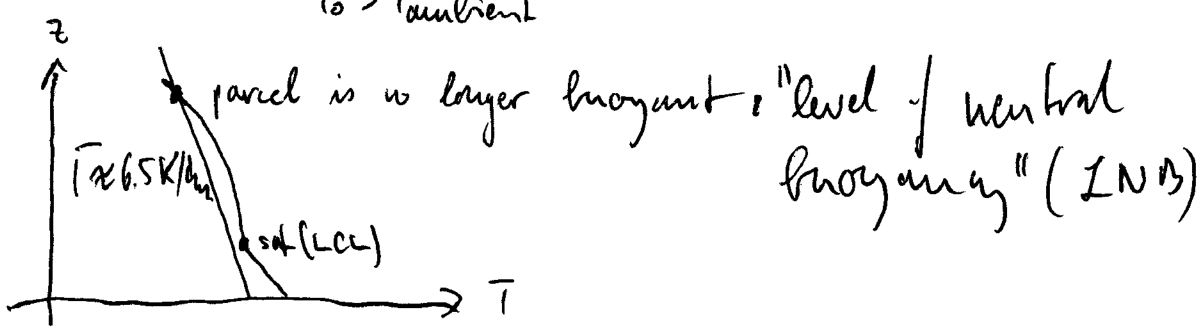




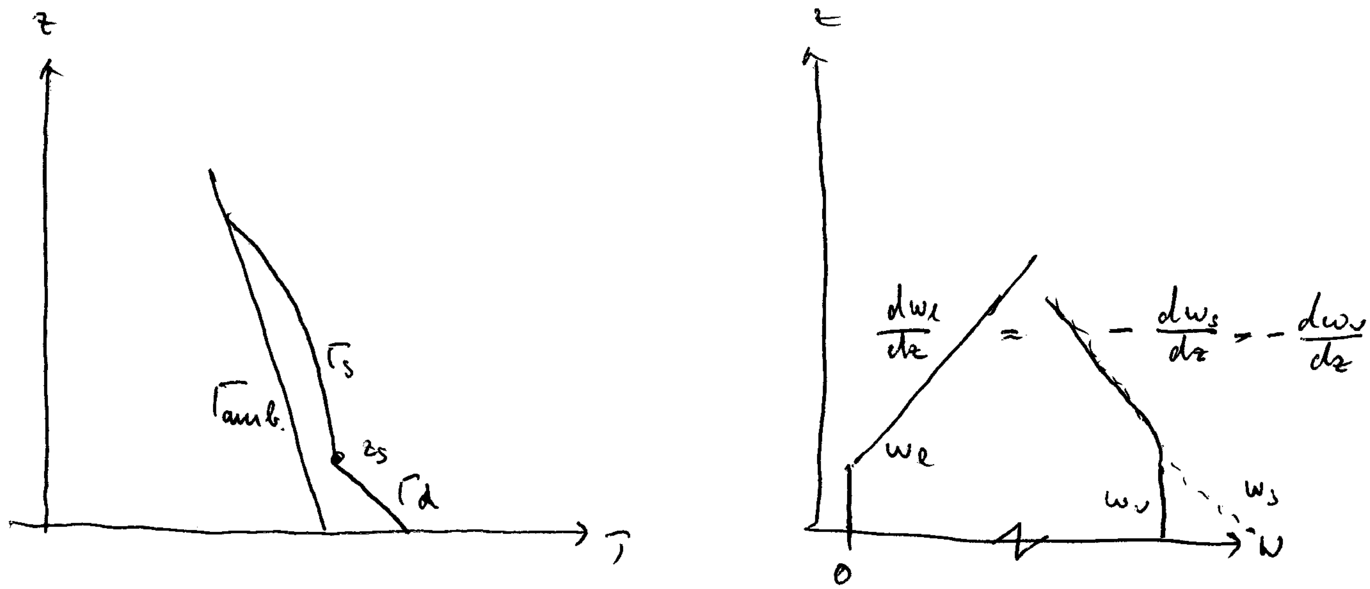
Role of the ambient lapse rate:



if entire atmosphere is at Γ_d, parcel continues forever...



Liquid water mixing ratio:

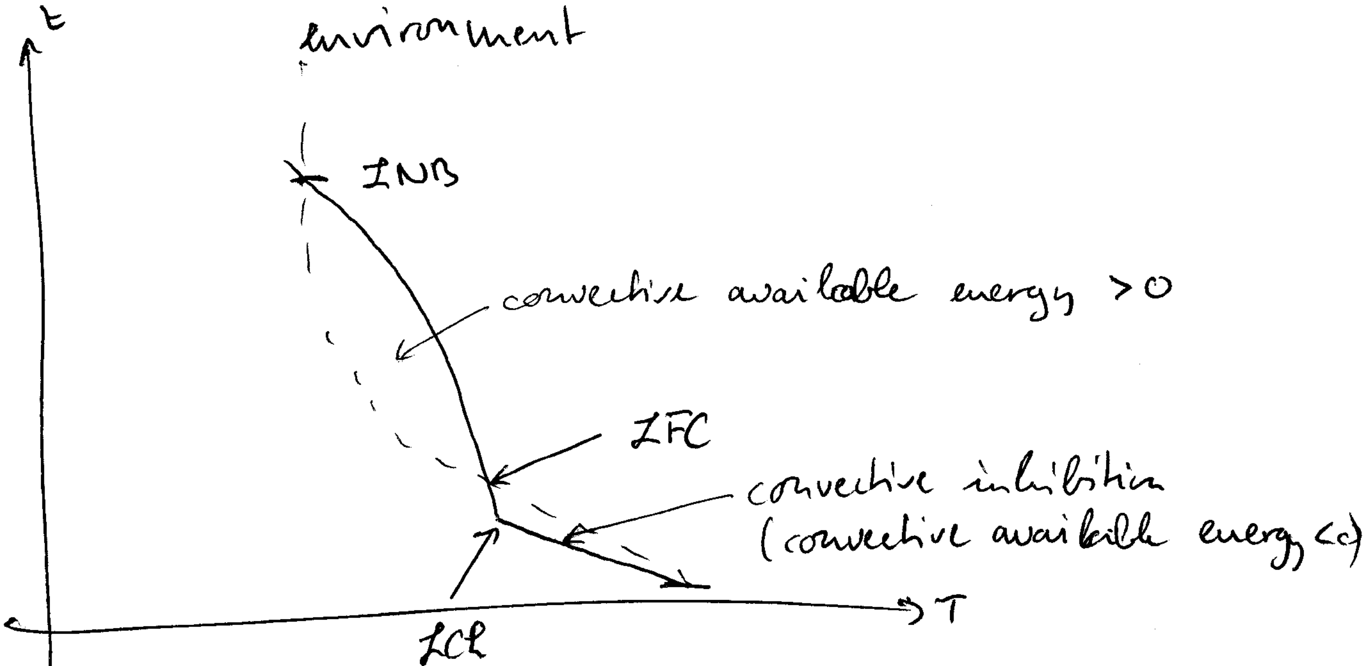


CAPE

How high can a parcel rise due to buoyancy? This is measured by the "convective available potential energy" (CAPE)

$$C(z) = \int_z^{Z_{NB}} g \, \frac{\rho - \rho'}{\rho'} \, dz'$$

If $C(z) > 0$, the parcel is able to ascend to the Z_{NB}; however, some amount of lifting may be required to reach the level of free convection (Z_{FC})

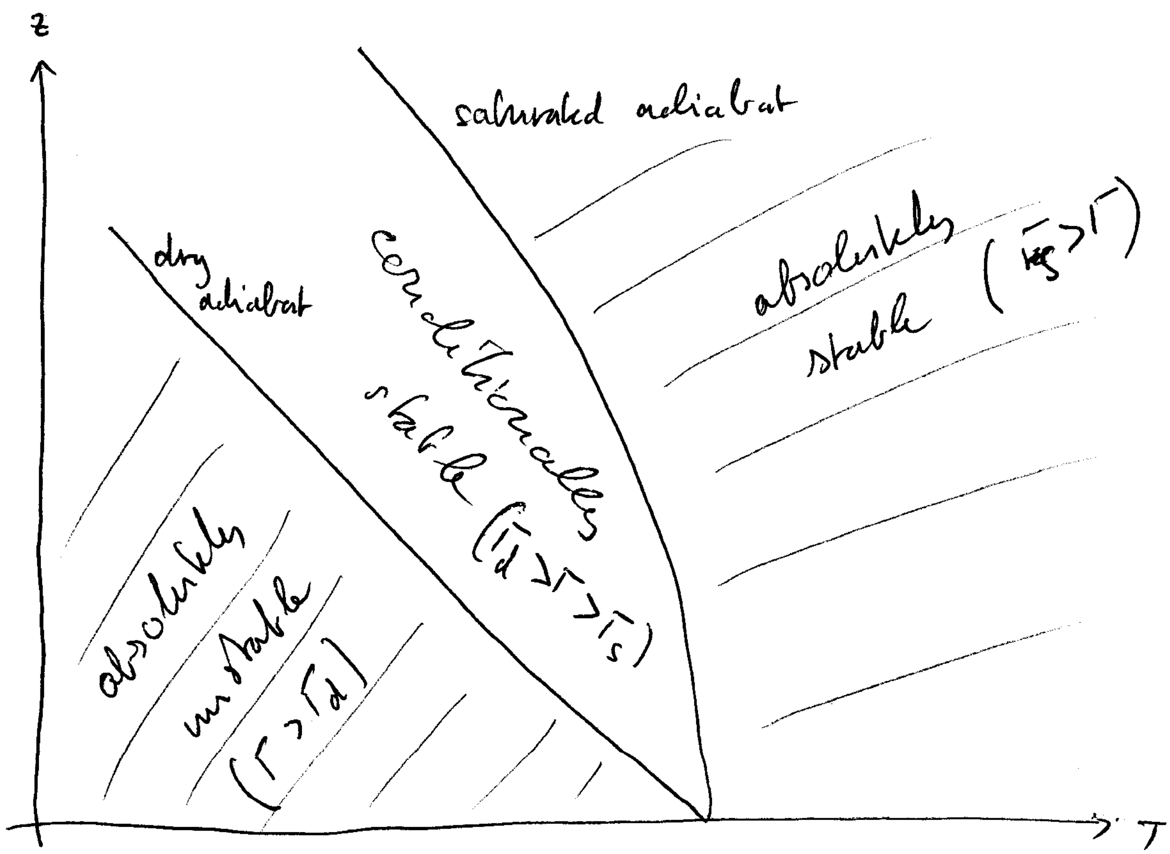


If $C(z) < 0$, parcel will not rise

In thunderstorms, CAPE ~ several 100 J/kg.

11/7 Stability

Suppose the environmental lapse rate is Γ (measured by weather balloon, aircraft arriving & departing from civilian airports, etc). Depending on the value of Γ, the atmosphere will either resist vertical motion (stable atmosphere) or not (unstable)



$\Gamma > \Gamma_d$: any air parcel, dry or wet, that is displaced upward will continue to rise

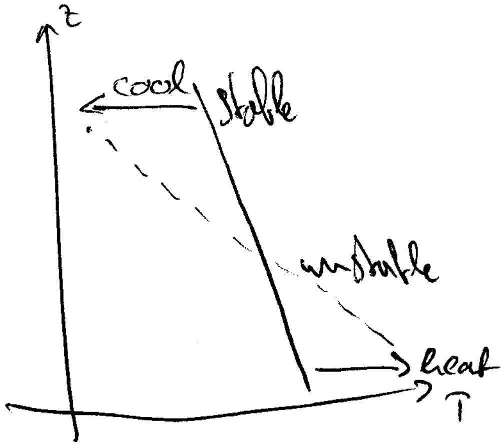
$\Gamma_d > \Gamma > \Gamma_{s}$: a dry air parcel will not continue to rise; a saturated parcel will continue to rise

$\Gamma_s > \Gamma$: any air parcel, dry or saturated, will stop rising

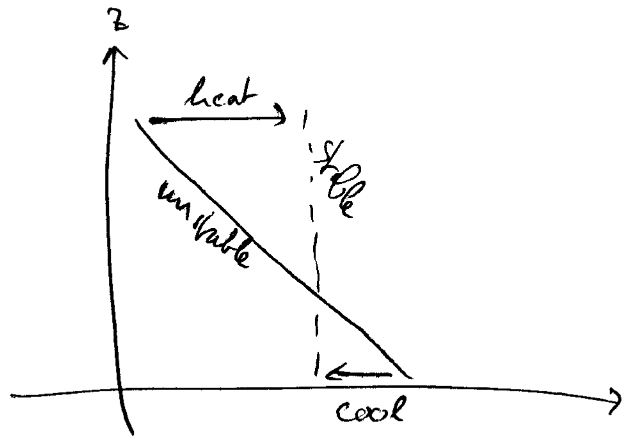
Processes modifying stability

Stability ~~of~~ is one of the most important properties of the atmosphere: an unstable atmosphere supports convection, leading to thunderstorms if the convection is strong enough; ~~an~~ ~~unsta~~ stable atmosphere can lead to stratification with low cloud bases or fog.

How can an unstable atmosphere be stabilized, or a stable atmosphere destabilized?



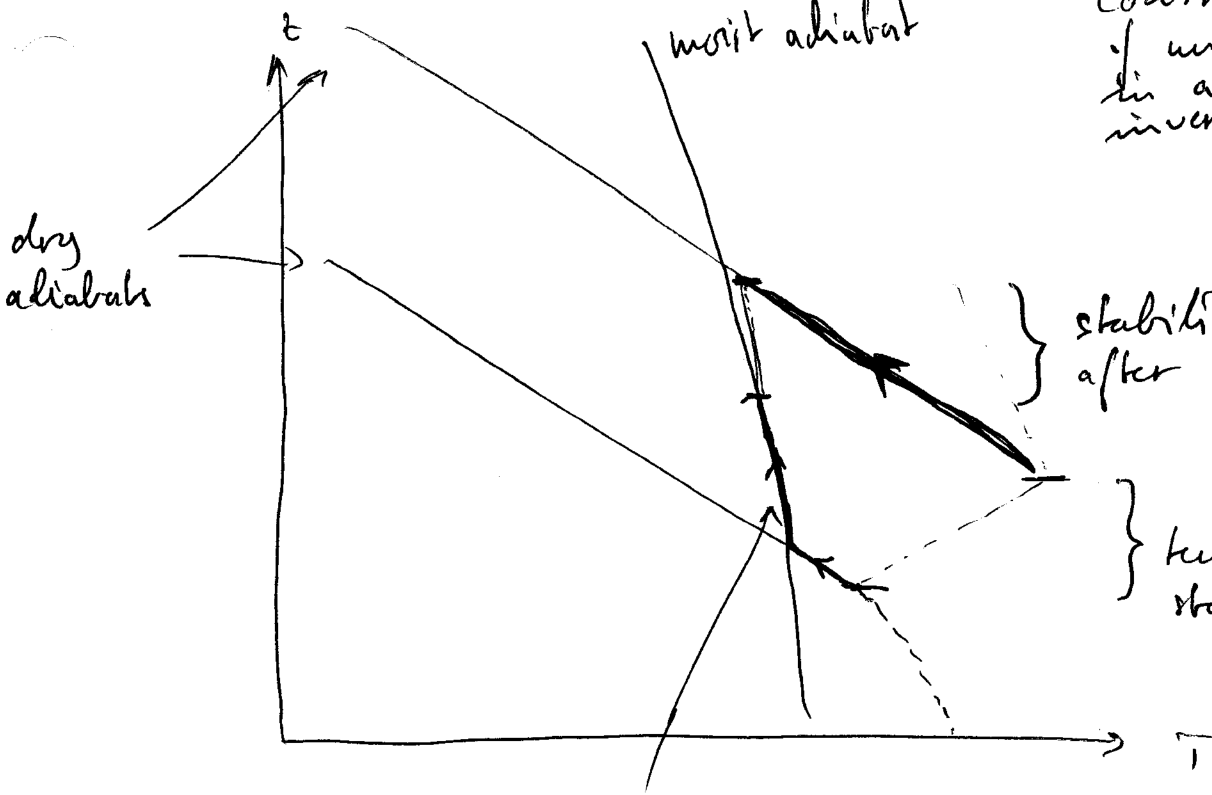
ex: warm tropical ocean
↳ convection



ex: cooling of surface at night
↳ ground fog

Destabilization by lifting:

Consider this layer of unsaturated air in a temperature inversion



stability is reduced after lifting

temp. inversion, stable

layer is mechanically lifted, for example by encountering a mountain