SIO 217a Atmospheric and Climate Sciences I: Atmospheric Thermodynamics

Fall 2014 Midterm Exam (No calculators, notes, books, PDAs.) **KEY**

Curry and Webster, Ch. 1-4 (and Section12.1)

Here are some numerical values, some of which may be useful on this exam:
Average radius of Earth: 6370 km
Mean reflectivity of the Earth: 0.31
Mean molecular weight of dry air: 29 g/mole
Mean molecular weight of water vapor: 18 g/mole
Gas constant for dry air, R_d: 287 J deg^{-1} kg^{-1}
Gas constant for water vapor, R_v: 461 J deg^{-1} kg^{-1}
Specific heat at constant pressure, c_p: 1004 J deg^{-1} kg^{-1}
Specific heat at constant volume, c_v: 717 J deg^{-1} kg^{-1}
Latent heat of vaporization for water at 273K, L_{lv}: 2.5×10^{6} J kg^{-1}
Solar luminosity: 3.92×10^{26} W
Earth-sun distance: 1.50×10^{11} m
Stefan-Boltzmann constant, σ: 5.67×10^{-8} W m^{-2} K^{-4}

1. Almost one-third of the Earth's incoming solar radiation is reflected back to space.
   a. Name the property of the Earth controls the fraction of incoming light reflected. Albedo is the mean reflectivity of the Earth.
   b. Calculate the amount of incoming solar radiation at the top of the atmosphere [in W m^{-2}].
   Instantaneous at solar noon =(luminosity)/(4pi*ESdistance^2)
   =(3.92×10^{26})/(4*3.14*(1.50×10^{11})^2)=1390 W m^{-2};
   Averaged over Earth surface = (1390 W m^{-2})/4=342 W m^{-2}.
   c. What happens to the energy from the incoming radiation that is not reflected? The remaining energy is absorbed by the Earth and the atmosphere and then re-emitted.
   d. If the amount of light reflected were *increased* by adding particles to the stratosphere, how would a simple model *with no greenhouse effect* predict Earth's temperature would respond? Give your model and state its assumptions.

Assume that: (1) the earth behaves as a blackbody, (2) atmosphere is transparent to non-reflected portion of the solar beam; (3) atmosphere in radiative equilibrium with surface. Then, at equilibrium, the incoming shortwave flux and outgoing longwave flux are equal (i.e. there is no accumulation) so for the normal solar luminosity we can write:

$F_L = \sigma T_{atm}^4$ (assumption 1; Eqn. 3.20)
$F_S = F_L$(assumption 2-3; Eqn. 3.20)
$0.25*S_0(1-\alpha_p) = \sigma T_{atm}^4$ (Eqn. 3.20, Eqn. 12.)
$T_{atm} = 255K$
where $S_0 = L_0/(4\pi d^2) = 1.3938\text{x}10^3$ W m^{-2} (Eqn. 12.), $\alpha_p=0$, $\sigma=5.67\times10^{-8}$ W m^{-2} K^{-4}
$\sigma T_{surf}^4 = 2*0.25*S_0(1-\alpha_p)$
As albedo increases, T_{surf} will decrease.

2. Consider the properties of the standard atmosphere, assuming hydrostatic balance and constant lapse rate of Γ=6.5K km^{-1}.

a. Write the equation for the hydrostatic balance and identify which two forces are "balanced" to give this equation.
b. Derive an expression for the variation of height with pressure ***z(p)***, in terms of the surface pressure p_0, surface temperature T_0 and a constant lapse rate Γ.
c. What are typical values for p_0 and T_0 for modern Earth?

a. ***dp=-ρgdz***: The hydrostatic balance is the equality of upward (pressure gradient) and downward (gravitational) forces in the atmosphere that results in little net vertical motion.

b. From the hydrostatic equation for an ideal gas (Eqn. 1.42)

$$\partial p = -\frac{pg}{R_d T}\partial z$$

and a constant lapse rate $\Gamma = -\frac{dT}{dz}$ we get

$$dp = -\frac{pg}{R_d T}dz$$

$$\frac{dp}{p} = -\left(\frac{g}{(T_0 - \Gamma z)R_d}\right)dz$$

$$\int_{p_0}^{p}\frac{dp}{p} = \int_0^z\left(-\frac{g}{R_d}\right)\frac{dz}{(T_0 - \Gamma z)}$$

$$\ln\frac{p}{p_0} = \left(\frac{g}{\Gamma R_d}\right)\ln\frac{(T_0 - \Gamma z)}{T_0}$$

$$\left(\frac{p}{p_0}\right)^{\left(R_d\Gamma/g\right)} = \left(\frac{T_0 - \Gamma z}{T_0}\right) = \left(1 - \frac{\Gamma z}{T_0}\right)$$

$$z = \frac{T_0}{\Gamma}\left[1 - \left(\frac{p}{p_0}\right)^{\left(R_d\Gamma/g\right)}\right]$$

c. p_0 =1013 mb; T_0= 288K.

3. Define the following terms in 10 words or less; an equation, graph, or sketch may be added if appropriate:
   a. phase equilibrium occurs when two substances have no net exchange of force, ($p_1=p_2$), temperature ($T_1=T_2$), or chemical potential ($\mu_1=\mu_2$).
   b. ideal gas is a vapor in which molecules collide with perfectly elastic collisions and without interactions, typically consistent with behavior at low pressures (<2 atm) and high temperatures (>250K); the equation of state is pv=RT.
   c. adiabatic means there is no exchange of heat into or out of the system; A path in which no heat is lost or gained during the process (Q=0).
   d. Stefan-Boltzmann law gives the irradiance of a black body as a function of temperature, i.e. $F=\sigma T^4$.
   e. reflectivity is the fraction of incoming radiation that is reflected.

f. entropy is a state function which gives the maximum (reversible) work done for a state change, given by $d\eta=(dq/T)_{rev}$ (Eqn. 2.25a)

g. virtual temperature is the temperature a parcel would have if there were no water vapor in it.

$$T_v = T(1+0.608q_v)$$

4. Give an equation for optical depth in terms of atmospheric properties. For a pathlength of 10 km, evaluate the optical depth for an average density of 1 kg m^{-3} and an average absorption coefficient of 3×10^{-5} m^2 kg^{-1}, what is the optical depth (assuming zenith angle of 0)?

$$d\tau_\lambda = k_\lambda^{abs}\rho\, dx \qquad (3.25)$$

Eqn. 3.25: $d\tau=k^{abs}\varrho dx$; so for average $k^{abs}\varrho$, $\tau=k^{abs}\varrho\Delta x$.
Optical depth $\tau=(3\text{x}10^{-5}\ m^2/kg)(1\ kg/m^3)(10000\ m)=0.3$

5. The saturation vapor pressure (of water) at a temperature of 20°C is 23.4 hPa. Consider moist air at 20°C, a pressure of 1000 hPa, and a relative humidity of 75%. Find the values:
   a. Vapor pressure
      $e=H*e_s = 0.75*23.4 = 17.6$ hPa [Eqn. 4.34a].
   b. mixing ratio
      $w_v=m_v/m_d=(M_v/M_d)*(e/(p-e))=0.622*(17.6/(1000-17.6))=0.0111$ [Eqn. 4.36].
   c. specific humidity
      $q_v=m_v/(m_v+m_d)=w_v/(w_v+1)=0.0110$ [Eqn. 1.20].
   d. potential temperature
      $\theta=T*(p_0/p)^{(Rd/cpd)} =T*(1000/1000)^{(Rd/cpd)}$ =293K or 20C
   e. saturation pressure (of water) at 30°C

$$e_2 = e_1 \exp\left[-\frac{L_{iv}}{R_v}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)\right] \qquad (4.26)$$

e_{30C}=(23.4 hPa)*exp[(-2.5e6/461)*(1/303-1/293)]~42 hPa

6. The saturation vapor pressure (of water) doubles approximately every 10°C in typical atmospheric conditions. This *strong dependence* of saturation vapor pressure on temperature (i.e. this large change of doubled pressure per 10°C increase in temperature) provides many of the unique cloud feedbacks that govern climate on Earth.

(a) Give the (i) first law of thermodynamics *in terms of internal energy* (and heat and work) and, (ii) in terms of *only other state variables and functions*, the equation that defines of enthalpy, entropy, and Gibbs free energy. (i) du=dq+dw; (ii) H=U+PV; $d\eta=dq_{rev}/T$; $G=H-T\eta$.

(b) For *a phase change* at equilibrium, what is (i) the change in Gibbs free energy, (ii) the change in enthalpy, and (iii) the change in entropy? Evaluate each in the simplest possible terms, if possible either an explicit value or a measured constant. (i) 0; (ii) L_{lv}; (iii) L_{lv}/T.

(c) Give *the differential form* of the equation that describes this increase in saturation vapor pressure with temperature. This equation is the Clausius-Clapeyron equation, and the differential form is given as:

$$\left(\frac{\partial p}{\partial T}\right)_g = \frac{L_{lv}\,p}{R_v T^2}$$

(d) Identify the water property *that appears in equation (a)* that makes this dependence strong. (Hint: The answer is a quantity related to water that is *not a pressure*.) Latent heat, L_{lv}, which is quite high for water.

7. The phase diagram of pure water is shown below. For these questions, assume that it describes (approximately) the three phases of water in the atmosphere.
   a. Label the triple point of water. H See A on diagram below.
   b. How many degrees of freedom are there when three phases of pure water coexist? 0.
   c. Label the region where liquid water exists with two degrees of freedom. See C on diagram below.
   d. Identify the region on the diagram where liquid and vapor coexist. See D on diagram below.

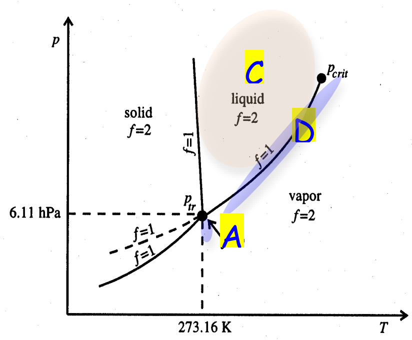

Figure 4.3 *p,T* phase diagram for water. The three curves indicate those points for which two phases coexist at equilibrium. The dashed curve is the extension of the vapor-pressure curve for liquid water to temperatures below 273.16 K. The solid curve below 273.16 K connects the points at which ice and vapor coexist at equilibrium. p_{crit} indicates the pressure and temperature values beyond which liquid water and water vapor are no longer distinguishable from one another. p_{tr} indicates the triple point, the unique *p,T* point at which all three phases coexist.