SIO 217a Atmospheric and Climate Sciences I: Atmospheric Thermodynamics

Fall 2013 Midterm Exam (No calculators, notes, books, PDAs.) **KEY**

Curry and Webster, Ch. 1-4 (and Section12.1)

Here are some numerical values, some of which may be useful on this exam:
Average radius of Earth: 6370 km
Mean reflectivity of the Earth: 0.31
Mean molecular weight of dry air: 29 g/mole
Mean molecular weight of water vapor: 18 g/mole
Gas constant for dry air, R_d: 287 J deg^{-1} kg^{-1}
Gas constant for water vapor, R_v: 461 J deg^{-1} kg^{-1}
Specific heat at constant pressure, c_p: 1004 J deg^{-1} kg^{-1}
Specific heat at constant volume, c_v: 717 J deg^{-1} kg^{-1}
Latent heat of vaporization for water at 273K, L_{lv}: 2.5×10^6 J kg^{-1}
Solar luminosity: 3.92×10^{26} W
Earth-sun distance: 1.50×10^{11} m
Stefan-Boltzmann constant, σ: 5.67×10^{-8} W m^{-2} K^{-4}

1. Almost one-third of the Earth's incoming solar radiation is reflected back to space.
   a. Name the property of the Earth controls the fraction of incoming light reflected. Albedo is the mean reflectivity of the Earth.
   b. Calculate the amount of incoming solar radiation at the top of the atmosphere [in W m^{-2}].
   Instantaneous at solar noon =(luminosity)/(4pi*ESdistance^2)
   =(3.92×10^{26})/(4*3.14*(1.50×10^{11})^2)=1390 W m^{-2};
   Averaged over Earth surface = (1390 W m^{-2})/4=342 W m^{-2}.
   c. What happens to the energy from the incoming radiation that is not reflected? The remaining energy is absorbed by the Earth and the atmosphere and then re-emitted.
   d. If the amount of light reflected were *decreased*, how would a simple model *with a perfectly absorbing greenhouse effect* predict Earth's temperature would respond? Give your model and state its assumptions.

Assume that: (1) the earth behaves as a blackbody, (2) atmosphere is transparent to non-reflected portion of the solar beam; (3) atmosphere in radiative equilibrium with surface; (4) atmosphere completely absorbs infrared radiation. Then, at equilibrium, the incoming shortwave flux and outgoing longwave flux are equal (i.e. there is no accumulation) so for the normal solar luminosity we can write:

$\mathbf{F_L = \sigma T_{atm}^4}$ (assumption 1; Eqn. 3.20)
$\mathbf{F_S = F_L}$(assumption 2-3; Eqn. 3.20)
$\mathbf{0.25*S_0(1-\alpha_p) = \sigma T_{atm}^4}$ (Eqn. 3.20, Eqn. 12.)
$\mathbf{T_{atm} = 255K}$
where $S_0 = L_0/(4\pi d^2)$ = 1.3938x10^3 W m^{-2} (Eqn. 12.), α_p=0, σ=5.67×10^{-8} W m^{-2} K^{-4}
$\mathbf{F_{surf} = 2F_{atm}}$ (assumption 4)
$\mathbf{\sigma T_{surf}^4 = 2\sigma T_{atm}^4}$
$\mathbf{\sigma T_{surf}^4 = 2*0.25*S_0(1-\alpha_p)}$
As albedo decreases, T_{surf} will increase.

2. Consider the properties of the standard atmosphere, assuming hydrostatic balance and constant lapse rate of Γ=6.5K km^{-1}.
   a. Write the equation for the hydrostatic balance and identify which two forces are "balanced" to give this equation.
   b. Derive an expression for the variation of height with pressure ***z(p)***, in terms of the surface pressure p_0, surface temperature T_0 and a constant lapse rate Γ.
   c. What are typical values for p_0 and T_0 for modern Earth?

a. $dp=-\rho g dz$: The hydrostatic balance is the equality of upward (pressure gradient) and downward (gravitational) forces in the atmosphere that results in little net vertical motion.
b. From the hydrostatic equation for an ideal gas (Eqn. 1.42)

$$\partial p = -\frac{pg}{R_d T}\partial z$$

and a constant lapse rate $\Gamma = -\frac{dT}{dz}$ we get

$$dp = -\frac{pg}{R_d T}dz$$

$$\frac{dp}{p} = -\left(\frac{g}{(T_0 - \Gamma z)R_d}\right)dz$$

$$\int_{p_0}^{p}\frac{dp}{p} = \int_0^z\left(-\frac{g}{R_d}\right)\frac{dz}{(T_0-\Gamma z)}$$

$$\ln\frac{p}{p_0} = \left(\frac{g}{\Gamma R_d}\right)\ln\frac{(T_0-\Gamma z)}{T_0}$$

$$\left(\frac{p}{p_0}\right)^{\left(R_d\Gamma/g\right)} = \left(\frac{T_0-\Gamma z}{T_0}\right) = \left(1-\frac{\Gamma z}{T_0}\right)$$

$$z = \frac{T_0}{\Gamma}\left[1-\left(\frac{p}{p_0}\right)^{\left(R_d\Gamma/g\right)}\right]$$

c. p_0 =1013 mb; T_0= 288K.

3. Define the following terms in 10 words or less; an equation, graph, or sketch may be added if appropriate:
   a. chemical equilibrium occurs when two substances have no net exchange of molecules from one phase to another (or from one species to another), i.e. $\mu_1=\mu_2$.
   b. ideal gas is a vapor in which molecules collide with perfectly elastic collisions and without interactions, typically consistent with behavior at low pressures (<2 atm) and high temperatures (>250K); the equation of state is pv=RT.
   c. virtual temperature The temperature of dry air having the same values of p and v as the moist air under consideration: $T_v=(1+0.608q_v)*T$ [Eqn. 1.25].
   d. Stefan-Boltzmann law gives the irradiance of a black body as a function of temperature, i.e. $F=\sigma T^4$.

e. absorptivity is the fraction of incoming radiation that is absorbed.
f. reversible work is work done in a process that proceeds with infinitesimally small steps that can be reversed at any step; it is equivalent to expansion work dw=-pdv.
g. Gibbs' phase rule says that for a system in thermal, chemical, and mechanical equilibrium, then the number of degrees of freedom are given by the Gibbs phase rule: $f=\chi-\varphi+2$ (for χ components in φ phases).

4. Give an equation for optical depth in terms of atmospheric properties. For a pathlength of 10 km, evaluate the optical depth for an average density of 1 kg m^{-3} and an average absorption coefficient of 3×10^{-5} m^2 kg^{-1}, what is the optical depth (assuming zenith angle of 0)?

$$d\tau_\lambda = k_\lambda^{abs} \rho \, dx \qquad (3.25)$$

Eqn. 3.25: $d\tau=k^{abs}\varrho dx$; so for average $k^{abs}\varrho$, $\tau=k^{abs}\varrho\Delta x$.
Optical depth $\tau=(3\times10^{-5}\ m^2/kg)(1\ kg/m^3)(10000\ m)=0.3$

5. The saturation vapor pressure (of water) at a temperature of 30°C is 42.4 hPa. Consider moist air at 30°C, a pressure of 1000 hPa, and a relative humidity of 75%. Find the values:
a. vapor pressure
$e=H*e_s = 0.25*42.4 = 10.6$ hPa [Eqn. 4.34a].
b. mixing ratio
$w_v=m_v/m_d=(M_v/M_d)*(e/(p-e))=0.622*(10.6/(1000-10.6))=0.00666$ [Eqn. 4.36].
c. specific humidity
$q_v=m_v/(m_v+m_d)=w_v/(w_v+1)=0.00662$ [Eqn. 1.20].
d. virtual temperature
$T_v=(1+0.608q_v)*T=(1+0.608q_v)*T=304.2K$ [Eqn. 1.25].
e. saturation pressure (of water) at 20°C

$$e_2 = e_1 \exp\left[-\frac{L_{iv}}{R_v}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)\right] \qquad (4.26)$$

$e_{20C}=(42.4\ hPa)*\exp[(-2.5e6/461)*(1/293-1/303)]$

6. The saturation vapor pressure (of water) doubles approximately every 10°C in typical atmospheric conditions. This *strong dependence* of saturation vapor pressure on temperature (i.e. this large change of doubled pressure per 10°C increase in temperature) provides many of the unique cloud feedbacks that govern climate on Earth.
(a) Give *the differential form* of the equation that describes this increase in saturation vapor pressure with temperature. This equation is the Clausius-Clapeyron equation, and the differential form is given as:

$$\left(\frac{\partial p}{\partial T}\right)_g = \frac{L_{lv} p}{R_v T^2}$$

(b) State the laws and assumptions used to derive the equation in (a). The laws and assumptions required are: (1) Definition of free energy and its properties as a state function; (2) Constant free energy of phase changes at constant temperature and pressure; (3) phase change from liquid to gas such that $v_L << v_V$; (4) ideal gas law for vapor, $v_V = R_V T/p$.

(c) Identify the water property *that appears in equation (a)* that makes this dependence strong. (Hint: The answer is a quantity related to water that is *not saturation vapor pressure*.) Latent heat, L_{lv}, which is quite high for water.

7. "Mixed phase" clouds (such as those found frequently in the Arctic) include water vapor as well as liquid, ice, or both. Typical temperatures for these mixed phase clouds are -40 to 0°C. In this question, you are asked to apply your knowledge of phase equilibrium to the behavior of water.
   a. If the pure water phase diagram below applies to a "warm" cloud, label the region of temperature and pressure where liquid and vapor coexist, *at equilibrium*, assuming pure water. **Label this A.** See diagram below.
   b. If the pure water phase diagram below applies to a "cold" cloud, label the region of temperature and pressure where ice and vapor coexist, *at equilibrium*, assuming pure water. **Label this B.** See diagram below.
   c. If the pure water phase diagram below applies to a "mixed" cloud, label the region of temperature and pressure where ice and liquid clouds coexist, *at equilibrium*, assuming pure water. **Label this C.** See diagram below, assuming that there is not vapor too (triple point is correct if assume vapor also present).
   d. If the pure water phase diagram below applies to this cloud, and liquid, ice, and vapor are all present, what is the temperature? **Label this D.** See diagram below.

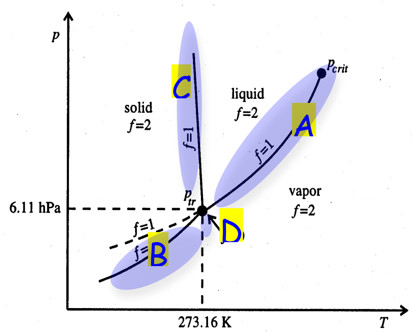

Figure 4.3 *p,T* phase diagram for water. The three curves indicate those points for which two phases coexist at equilibrium. The dashed curve is the extension of the vapor-pressure curve for liquid water to temperatures below 273.16 K. The solid curve below 273.16 K connects the points at which ice and vapor coexist at equilibrium. p_{crit} indicates the pressure and temperature values beyond which liquid water and water vapor are no longer distinguishable from one another. p_{tr} indicates the triple point, the unique *p,T* point at which all three phases coexist.