Course Principles

- Green classroom
  - Minimal handouts, optional paper text, etc.
- Respect for learning
  - On time, on schedule: quizzes
  - No chatting (in class), no cheating
- Focused exams
  - Core principles not algebra
- "Office hours" – help on homework, projects
- Team learning by projects
  - Bring different backgrounds to common topics

Homework Schedule

Email Single PDF to lmrussell@ucsd.edu

- Due Oct. 14 (Monday, 12 noon)
  - Ch. 1, Problem 11
  - Ch. 2, Problem 2
- Due Oct. 23 (Wednesday, 12 noon)
  - Ch. 3, Problem 1, 2 (hypo in answer key), 3
- Due Oct. 30 (Wednesday, 12 noon)
  - Ch. 4, Problem 4, 5
- Midterm Nov. 6 (Wednesday, in class)
- Due Nov. 13 (Wednesday, 12 noon)
  - Ch. 5, Problem 3, 7 (erratum in 7d)
  - Ch. 6, Problem 4, 6
- Due Nov. 18 (Monday, 12 noon)
  - Ch. 7, Problem 3 (not graded, outline approach only, discuss)

What do we learn in Ch. 1?

- What P, T, U are for a fluid
- What an ideal gas is
- How P, T, v relate for an ideal gas (and we call this relationship an equation of state)
- What chemical components constitute the atmosphere (for homosphere <110 km)
- What the hydrostatic balance is
- How p, T vary with z for observed, "standard," isopycnic, isothermal, constant lapse-rate atmospheres
The Greenhouse Effect

Solar radiation

Long-wave radiation

Review Topics in Ch. 1

- Thermodynamic quantities
- Composition
- Pressure
- Density
- Temperature
- Kinetic Theory of Gases

IPCC 2013

Atmospheric Carbon Dioxide
Measured at Mauna Loa, Hawaii

IPCC 2013

Cumulative total anthropogenic CO₂ emissions from 1870 (GtCO₂)

Cumulative total anthropogenic CO₂ emissions from 1870 (GtCO₂)

Thermal Structure of the Atmosphere

Curry and Webster, pp. 1-17
Feynman, Book I, ch. 39
Thermodynamic Quantities

- Classical vs. Statistical thermodynamics
- Open/closed systems
- Equation of state \( f(P,V,T)=0 \)
- Extensive/intensive properties
- Thermal, engine, heat/work cycles

Composition

- Structure
  - Comparison to other planets
- \( \text{N}_2, \text{O}_2, \text{Ar}, \text{CO}_2, \text{H}_2\text{O} \): 110 km constitute 99%
- Water, hydrometeors, aerosol

Pressure

- Force per unit area
- 1 bar = 10^5 Pa; 1 mb = 1 hPa; 1 atm = 1.013 bar
- Atmosphere vs. Ocean

<p>| Table 1.1 Main gaseous constituents of air, relative to the percent composition of dry air. |
|----------------------------------|-----------------|-------------|-------------|</p>
<table>
<thead>
<tr>
<th>Constituent</th>
<th>Formula</th>
<th>Molecular weight</th>
<th>% by volume</th>
<th>% by mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrogen</td>
<td>( \text{N}_2 )</td>
<td>28.016</td>
<td>78.08</td>
<td>75.51</td>
</tr>
<tr>
<td>Oxygen</td>
<td>( \text{O}_2 )</td>
<td>31.999</td>
<td>20.95</td>
<td>23.14</td>
</tr>
<tr>
<td>Argon</td>
<td>( \text{Ar} )</td>
<td>39.948</td>
<td>0.93</td>
<td>1.28</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>( \text{CO}_2 )</td>
<td>44.010</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>Water vapor</td>
<td>( \text{H}_2\text{O} )</td>
<td>18.005</td>
<td>0.4</td>
<td></td>
</tr>
</tbody>
</table>
Pressure

- Force per unit area
  - 1 bar = 10^5 Pa
  - 1 mb = 1 hPa
  - 1 atm = 1.013 bar

Density

- Specific volume: \( v = \frac{V}{m} \)
  - 0.78 m^3 kg^{-1} for air
- Density: \( \rho = \frac{m}{V} \)
  - 1.29 kg m^{-3} for air

Temperature

- "Zeroeth" Law of Thermodynamics
  - Equilibrium of two bodies with third
  - Allows universal temperature scale
- Temperature scale
  - Two fixed points: Kelvin, Celsius
  - Thermometer
- Lapse Rate \( \Gamma = -\frac{dT}{dz} \)
  - Change in temperature with altitude
    - Typically \( \Gamma \approx 6.5 \text{ K/km} \)
- Temperature inversion \( \Gamma < 0 \)
  - Boundary layer "cap"
  - Tropopause between troposphere and stratosphere

History of the Standard Atmosphere

- With a little digging, you can discover that the Standard Atmosphere can be traced back to 1920. The constant lapse rate of 6.5° per km in the troposphere was suggested by Prof. Toussaint, on the grounds that...
  - ...what is needed is... merely a law that can be conveniently applied and which is sufficiently in accordance with the means adhered to. By this method, corrections due to temperature will be as small as possible in calculations of airplane performance, and will be easy to calculate...
  - The deviation is of some slight importance only at altitudes below 1,000 meters, which altitudes are of little interest in aerial navigation. The simplicity of the formula largely compensates...
- The above quotation is from the paper by Gregg (1920). The early motivations for this simplified model were evidently the calibration of aneroid altimeters for aircraft, and the construction of firing tables for long-range artillery, where air resistance is important.
- Unfortunately, it is precisely the inaccurate region below 1,000 m that is most important for refraction near the horizon. However, the Toussaint lapse rate, which Gregg calls "arbitrary," is now embodied in so many altimeters that it cannot be altered: all revisions of the Standard Atmosphere have preserved it. Therefore, the Standard Atmosphere is really inappropriate for astronomical refraction calculations. A more realistic model would include the diurnal changes in the boundary layer, but these are still so poorly understood that no satisfactory basis seems to exist for realistic refraction tables near the horizon.

International Standard Atmosphere

- The ISA model divides the atmosphere into layers with linear temperature distributions. The other values are computed from basic physical constants and relationships. Thus the standard consists of a table of values at various altitudes, plus some formulas by which those values were derived. For example, at sea level the standard gives a pressure of 1.013 bar and a temperature of 15°C, and an initial lapse rate of -6.5°C/km. Above 12 km the calculated temperature is essentially constant. The tables continue to 18 km where the pressure has fallen to 0.085 bar and the temperature to -56.5°C.

Geopotential Height

Geopotential height is a vertical coordinate referenced to Earth's mean sea level. It is an approximation to geopotential height (a geopotential height above mean sea level) using the variation of gravity with latitude and elevation. This can be considered a "geodetic" height. One usually speaks of the geopotential height of a certain pressure level, which would correspond to the geopotential height necessary to reach the given pressure.

At an acceleration of \( a \), the geopotential is defined as:

\[ \Phi = \int_0^h g \Phi \, dt, \]

where \( g \) is the acceleration due to gravity, \( \Phi \) is latitude, and \( h \) is geopotential elevation.

Thus, the geopotential is the potential energy per unit mass at this level. The geopotential height is:

\[ Z = \Phi \]

where \( Z \) is the geopotential height at a standard gravity on mean sea level.

Geophysical science often uses geopotential height rather than geopotential height. Because doing so in many cases makes analytical calculations more convenient. For example, the primitive equations which describe forecast models are more easily expressed in terms of geopotential than geopotential height. Using this form eliminates vertical force on air density (which is very difficult to measure) in the equations.
ICAO Standard Atmosphere

The International Civil Aviation Organization (ICAO) Standard Atmosphere gives the average values for meteorological elements at 40°N from mean sea level (MSL) to 90 km (292,000 ft).

The ICAO Standard Atmosphere does not contain water vapour.

Some of the values defined by ICAO are:

<table>
<thead>
<tr>
<th>Height km &amp; 0</th>
<th>Temperature °C</th>
<th>Pressure hPa</th>
<th>Lapse Rate °C/1000 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>0km MSL</td>
<td>15.0</td>
<td>1013.25</td>
<td>1.96 (Troposphere)</td>
</tr>
<tr>
<td>11km 30,000ft</td>
<td>56.5</td>
<td>220.00</td>
<td>0.00 (Stratospheric)</td>
</tr>
<tr>
<td>24km 65,000ft</td>
<td>56.5</td>
<td>54.70</td>
<td>-1.00 (Stratospheric)</td>
</tr>
<tr>
<td>32km 105,000ft</td>
<td>44.5</td>
<td>8.68</td>
<td></td>
</tr>
</tbody>
</table>

As this is a Standard, you will never encounter these conditions outside of a laboratory, but many Aviation Standards and flying rules are based on this, altitude being a major one. The standard is very useful in Meteorology for comparing actual values to.

Hydrostatic Balance

• Applicable to most atmospheric situations (except fast accelerations in thunderstorms)

\[ g = -\frac{1}{\rho} \frac{\partial \rho}{\partial z} \]

• Why? This is just a force balance on an air parcel.

\[ g = -A \left( \frac{\partial \rho}{\partial z} \right) = -\frac{\partial p}{\partial z} \]

Curry and Webster, Ch. 1

Homogeneous Atmosphere

• Density is constant
• Surface pressure is finite
• Scale height H gives where pressure=0

\[ H = \frac{R T_0}{g} \]

\[ \rho = \rho_0 \left( \frac{p_0}{p} \right) \]

\[ \int_{p_0}^{p} d\rho = \int H \rho_0 dH \]

\[ \rho_c \equiv \text{constant} \]

\[ p_0 = \rho g H \]

\[ \frac{\partial p}{\partial z} = \rho R \frac{\partial T}{\partial z} \]

\[ g = -\frac{1}{\rho} \frac{\partial \rho}{\partial z} \]

N.B. ideal gas

\[ \Gamma = -\frac{\partial T}{\partial z} = \frac{g}{R} = 34.1 \text{°C/km} \]

Curry and Webster, Ch. 1

Hydrostatic Equation (1)

• Hydrostatic Balance (1.33)

\[ g = -\frac{1}{\rho} \frac{\partial \rho}{\partial z} \]

• Geopotential Height (1.36a)

\[ Z = -\frac{1}{g} \int g dz \]

• Homogeneous atmosphere (1.38)

\[ p_c = \rho g H \]  
[N.B. \( \rho \equiv \text{constant} \)]  
\[ \frac{\partial p}{\partial z} = \rho R \frac{\partial T}{\partial z} \]  
[N.B. ideal gas]

\[ H = \frac{R T_c}{g} = 8 \text{ km} \]

\[ \Gamma = -\frac{\partial T}{\partial z} = \frac{g}{R_c} = 34.1 \text{°C/km} \]
Hydrostatic Equation (2)

- Isothermal Atmosphere (1.42)
  \[ \frac{dp}{dt} = \frac{-g}{R_T} \frac{dT}{dz} \]
  [N.B. \( T \) = constant]
  \[ p = p_0 \exp \left( \frac{-z}{H} \right) \] for \( H = \frac{RT}{g} \)

- Constant Lapse Rate (1.48)
  \[ \frac{dp}{p} = \frac{g}{R_T} \frac{dz}{z_T} \]
  [N.B. \( \Gamma \) = constant]
  \[ p = p_0 \left( \frac{T_0}{T} \right)^{\Gamma} \]

Kinetic Theory of Gases

- What is the pressure of a gas?
- What is the temperature of a gas?
- Pressure-volume-temperature relationship(s)
- How does pressure (and volume) relate to energy?
  - Kinetic energy
  - Internal energy
    - The “fine print”

If all atoms had same \( x \)-velocity \( v_x \):
- Momentum Change for one Atom-Collision: [Initial] - [Final] = \( mv_x \) - (\( -mv_x \)) = \( 2mv_x \)
- Number of Atom-Collisions-Per-Time: [Concentration] [Volume] = \( [n] [v_x A] \)
- Force = [Number] [Momentum Change] = \( [nv_x A] [2mv_x] \) = \( 2nmAv_x^2 \)
- Pressure = [Force] / [Area] = \( 2nmv_x^2 \)

For atoms with average velocity-squared of \( <v_x^2> \):
- Pressure = [Force] / [Area] = \( nm<v_x^2> \)

Population-averaged Velocity: \( <v_x^2> = v_x^2 + v_y^2 + v_z^2 \)\( \bar{n} \)
- Scalar multipliers: \( <mv_x^2>/2 = <v_x^2> + v_y^2 + v_z^2 \)
- How many will hit “right” wall? \( \bar{n}/2 \)

3D velocity: \( <v^2> = <v_x^2> + <v_y^2> + <v_z^2> \)
- Random motion (no preferred direction): \( <v_x^2> = <v_y^2> = <v_z^2> \)
- \( <v_x^2> = <v^2>/3 \)

\( P = \frac{nmv_x^2}{2} \)
- Total “internal” energy: \( U = \frac{1}{2} \bar{n} v_x^2 \)
- Kinetic energy of gas: \( U = \frac{1}{2} \bar{n} v_x^2 \)
- \( \frac{2}{3} PU \)

Concentration: \( n/NV \)
- Total “internal” energy: \( U = \frac{2}{3} \bar{n} E_k \)
- Kinetic energy of gas
\[ PV = \left\{ \frac{2}{3} \right\} E_k \]

\[ E_k = \left\{ \frac{3}{2} \right\} PV \]

Define \( T = \frac{1}{\beta N} \)

For scale choose \( T = \frac{2}{3} \frac{E_k}{Nk} \)

\[ E_k = \left\{ \frac{3}{2} \right\} NkT \]

Then \( PV = NkT = nR*T \)

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Temperature is defined to be proportional to the average kinetic energy of the molecules.

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Kinetic energy of gas

RHS is independent of gas

\( \rightarrow \) so scale can be universal

\[ \frac{E}{Nk} = \left\{ \frac{3}{2} \right\} kT \]

\[ k = 1.38 \times 10^{-23} \text{ J/K} \]

\[ R^* = Nj = 8.314 \text{ J mole/K} \]

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Diffusive Separation of the Lower Atmosphere

In 1926, John Dalton suggested that the composition of Earth's atmosphere could potentially change with altitude because of the separation of heavy and light molecules in Earth's gravitational field. Subsequent measurements, however, found convincing evidence of separation only above 100 km altitude, re-inforcing the view that any separation effect is overwhelmed by turbulent mixing at lower elevations. Here we show, by using precise measurements of the \( \frac{Ar}{N_2} \) ratio, that a dense layer separation can also occur in near-surface layers, although temperature gradients rather than gravity appear to be the main driving force. 

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