Course Principles

• Green classroom
  – Minimal handouts, optional paper text, etc.
• Respect for learning
  – On time, on schedule: quizzes
  – No chatting (in class), no cheating
• Focused exams
  – Core principles not algebra
• “Friday classes” – help on homework, projects
• Team learning by projects
  – Bring different backgrounds to common topics
Review from Ch. 1

- Thermodynamic quantities
- Composition
- Pressure
- Density
- Temperature
- Kinetic Theory of Gases

- Homework problem Ch. 2, Prob. 2 (due Monday 10/8; recitation on Friday 10/5)

Thermodynamic Quantities

- Classical vs. Statistical thermodynamics
- Open/closed systems
- Equation of state \( f(P,V,T)=0 \)
- Extensive/intensive properties
- Thermal, engine, heat/work cycles

Composition

- Structure
  - Comparison to other planets
- \( \text{N}_2, \text{O}_2, \text{Ar}, \text{CO}_2, \text{H}_2\text{O}: \) 110 km constitute 99%
- Water, hydrometeors, aerosol

| Table 1.1: Main gaseous constituents of air, relative to the percent composition of dry air |
|---------------------------------|--------|--------|--------|
| Constituent | Formula | Molecular weight | % by volume | % by mass |
| Nitrogen     | \( \text{N}_2 \) | 28.016 | 78.08 | 75.51 |
| Oxygen       | \( \text{O}_2 \) | 31.999 | 20.95 | 23.14 |
| Argon        | \( \text{Ar} \) | 39.948 | 0.93 | 1.28 |
| Carbon dioxide | \( \text{CO}_2 \) | 44.010 | 0.03 | 0.05 |
| Water vapor  | \( \text{H}_2\text{O} \) | 18.005 | 0.4 |
Pressure

• Force per unit area
  - 1 bar = $10^5$ Pa; 1 mb = 1 hPa; 1 atm = 1.013 bar
• Atmosphere vs. Ocean

Density

• Specific volume: $v = V/m$
  - 0.78 m$^3$ kg$^{-1}$ for air
• Density: $\rho = m/V$
  - 1.29 kg m$^{-3}$ for air

Temperature

• "Zeroth" Law of Thermodynamics
  - Equilibrium of two bodies with third
  - Allows universal temperature scale
• Temperature scale
  - Two fixed points: Kelvin, Celsius
  - Thermometer
• Lapse Rate $\Gamma = -\partial T/\partial z$
  - Change in temperature with altitude
  - Typically $\Gamma = 6.5$ K/km
• Temperature inversion $\Gamma < 0$
  - Boundary layer "cap"
  - Tropopause between troposphere and stratosphere

History of the Standard Atmosphere

• With a little digging, you can discover that the Standard Atmosphere can be traced back to 1920. The constant lapse rate of 6.5° per km in the troposphere was suggested by Prof. Toussaint, on the grounds that ...
• ...what is needed is ... merely a law that can be conveniently applied and which is sufficiently in concordance with the means adhered to, by this method, corrections due to temperature will be as small as possible in calculations of airplane performance, and will be easy to calculate ... The deviation is of some slight importance only at altitudes below 1,000 meters, which altitudes are of little interest in aerial navigation. The simplicity of the formula largely compensates this inconvenience.
• The above quotation is from the paper by Gregg (1920). The early motivations for this simplified model were evidently the calibration of aneroid altimeters for aircraft, and the construction of firing tables for long-range artillery, where air resistance is important.
• Unfortunately, it is precisely the inaccurate region below 1000 m that is most important for refraction near the horizon. However, the Toussaint lapse rate, which Gregg calls "arbitrary", is now embodied in so many altimeters that it cannot be altered. All revisions of the Standard Atmosphere have preserved it.
• Therefore, the Standard Atmosphere is really inappropriate for astronomical refraction calculations. A more realistic model would include the diurnal changes in the boundary layer; but these are still so poorly understood that no satisfactory basis seems to exist for realistic refraction tables near the horizon.

http://mintaka.sdsu.edu/GF/explain/thermal/std_atm.html
International Standard Atmosphere

The ISA model divides the atmosphere into layers with linear temperature distributions.[2] The other values are computed from basic physical constants and relationships. Thus the standard consists of a table of values at various altitudes, plus some formulas by which those values were derived. For example, at sea level the standard gives a pressure of 1.013 bar and a temperature of 15°C, and an initial lapse rate of -6.5 °C/km. Above 12km the tabulated temperature is essentially constant. The tabulation continues to 18km where the pressure has fallen to 0.075 bar and the temperature to -56.5 °C.[3][4]

ICAO Standard Atmosphere

The International Civil Aviation Organization (ICAO) Standard Atmosphere gives the average values for meteorological element at 40°N from mean sea level (MSL) to 80km (522,900 ft).

The ICAO Standard Atmosphere does not contain water vapour

Some of the values defined by ICAO are:

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>Temperature (°C)</th>
<th>Pressure (hPa)</th>
<th>Lapse Rate (°C/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15.0</td>
<td>1013.25</td>
<td>1.00 (Tropospheric)</td>
</tr>
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<td>15.0</td>
<td>54.70</td>
<td>1.00 (Stratospheric)</td>
</tr>
<tr>
<td>22.2</td>
<td>11.0</td>
<td>44.5</td>
<td>0.976</td>
</tr>
<tr>
<td>32.2</td>
<td>11.0</td>
<td>10.4</td>
<td>0.976</td>
</tr>
</tbody>
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As this is a Standard, you will never encounter these conditions outside of a laboratory, but many aviation standards and flying rules are based on this, altimeter being a major one. The standard is very useful in Metrology for comparing actual values.

ICAO Standard Atmosphere

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Geopotential Height

Geopotential height is a vertical coordinate referenced to Earth’s mean sea level — an adiabatic to geopotential height relation above mean sea level using the variation of gravity with latitude and elevation. Thus it can be considered as gravity adjusted height. One usually speaks of the geopotential height of a certain pressure level, which would correspond to the geopotential height anomaly to reach the given pressure.

At an elevation of h, the geopotential is defined as:

\[ \Phi = \int g(z) \, dz \]

where \( g(z) \) is the acceleration due to gravity, \( z \) is latitude, and \( h \) is the geopotential elevation.

Then, it is the gravitational potential energy per unit mass at that level. The geopotential height is

\[ Z_g = \frac{\Phi}{g} \]

where \( Z_g \) is the standard gravity at mean sea level.

Geophysical scientists often use geopotential height rather than geopotential because doing so in many cases makes analytical calculations more convenient. For example, the primitive equations (which render force needed solve are more easily expressed in terms of geopotential than geopotential height). Using the above equations obtains satisfying force and so density which is very difficult to measure in the atmosphere.

Homogeneous Atmosphere

- Density is constant
- Surface pressure is finite
- Scale height \( H \) gives where pressure = 0

\[ \Gamma = \frac{\partial p}{\partial z} = \frac{g}{R_T} \]

\[ Z_s = \frac{\Phi}{g} \]

Hydrostatic + Ideal Gas + Homogeneous

- Evaluate lapse rate by differentiating ideal gas law
**Lapse Rate**

- \( \Gamma = -\frac{dT}{dz} \)
- Change in temperature with height
- Defined to be positive in troposphere

Troposphere: \( \frac{dT}{dz} = 70 \text{ C} \)
\dz = 10 \text{ km}

So, \( \Gamma = 7 \text{ C/km} \)

**Hydrostatic Equation (1)**

- Hydrostatic Balance (1.33)
\[ g = -\frac{1}{\rho} \frac{\partial p}{\partial z} \]

- Geopotential Height (1.36a)
\[ Z = -\frac{1}{g_0} \int g_z dz \]

- Homogeneous atmosphere (1.38)
\[ \rho = \rho g H \]
\[ \frac{\partial p}{\partial z} = \rho R_d \frac{\partial T}{\partial z} \]
\[ \Gamma = -\frac{\partial T}{\partial z} = g \frac{R_d}{R} = 34.1 \text{ C km}^{-1} \]

**Hydrostatic Equation (2)**

- Isothermal Atmosphere (1.42)
\[ \frac{dp}{\rho} = \frac{R}{\rho g} \frac{dT}{dz} \]
\[ \text{N.B.} \ T = \text{constant} \]
\[ p = p_0 \exp\left(-\frac{z}{H}\right) \text{ for } H = \frac{RT}{g} \]

- Constant Lapse Rate (1.48)
\[ \frac{dp}{\rho} = -\frac{g}{R_d} \frac{\partial T}{\partial z} \]
\[ \text{N.B.} \ \Gamma = \text{constant} \]
\[ p = p_0 \left(\frac{T}{T_0}\right)^{\frac{R}{R_d}} \]

**Kinetic Theory of Gases**

- What is the pressure of a gas?
- What is the temperature of a gas?
- Pressure-volume-temperature relationship(s)
- How does pressure (and volume) relate to energy?
- Kinetic energy
- Internal energy — The "fine print"
Population-averaged Velocity: $<v^2>=v_i^2 + v_j^2 + \ldots + v_n^2/n$
Scalar multipliers: $<mv^2>/2=n(mv_i^2 + mv_j^2 + \ldots + mv_n^2)/2n$

How many will hit "right" wall? $n/2$

3D velocity: $<v^2>=<v_x^2>+<v_y^2>+<v_z^2>$
Random motion (no preferred direction): $<v_x^2>=<v_y^2>=<v_z^2>$

$<v^2>/3=2/3\text{[kinetic energy of molecule]}$

$\text{PV}=[2/3]\text{[E_k]}$
$\text{E}_n=[2/3]\text{[PV]}$

Define $T=E_k/(3/2)Nk$

$\text{RHS is independent of gas}$

--> so scale can be universal

Mean k.e.: $E_k/N=(3/2)kT$

$k=1.38x10^{-23}\text{[J/K]}$

$k=1.38x10^{-23}\text{[J/K]}$

$T^*=Nk=8.314\text{[J/mole*K]}$