The atmosphere absorbs part of the outgoing longwave energy

Incoming solar radiation

Outgoing reflected energy $F_r \sim (\alpha_p \sim 31\%)$

The White House Effect

$\text{CO}_2 + \text{Aerosols}$

Outgoing absorbed energy $F_I$ (infrared)

The Greenhouse Effect

Solar Radiation

- Solar radiation flux at the average distance of Earth’s orbit ($S_0$) is about 1367 W m$^{-2}$
  (This is equivalent to almost 23 60W light bulbs every square meter)
- This solar radiation intersects with $\pi R^2$ area, where $R$ is the radius of the Earth

Solar Radiation

- The total energy rate for solar radiation intersecting Earth is $S_0 \pi R^2$
- The global surface area of Earth is $4\pi R^2$
- Divide the total energy rate by the global surface area to obtain the global average solar radiation flux at the top of the atmosphere: $S_0 / 4 = 342$ W m$^{-2}$

Planetary Albedo

- Planetary albedo $\alpha_p$ is the global average fraction of solar radiation reflected back to space.
  - Earth’s planetary albedo is 0.31
  - The global average value of reflected solar flux is $\alpha_p S_0 / 4 = 107$ W m$^{-2}$
  - The global average value of absorbed solar flux is $(1 - \alpha_p) S_0 / 4 = 235$ W m$^{-2}$

The Simplest Model

- The Earth absorbs solar radiation $S_0 / 4$
- The Earth has a certain planetary albedo for solar wavelengths $(1 - \alpha_p) S_0 / 4$
- The Earth emits like a blackbody at IR wavelengths $\sigma T_e^4$

The Simplest Model

- The Earth has no atmosphere $T_e$ is the surface temperature
- The Earth is in radiative balance $(1 - \alpha_p) S_0 / 4 = \sigma T_e^4$
The Simplest Model

We now have our simple model of Earth’s climate:

\[(1 - \alpha_p) S_0 / 4 = \sigma T_e^4\]

What is the value of \(T_e\)?

Rearrange the equation so that \(T_e\) is on the left side and everything else is on the right side

\[T_e = \left(\frac{(1 - \alpha_p) S_0}{4\sigma}\right)^{1/4}\]

A Simple Atmosphere

• Now add an atmosphere to the model
• This simple atmosphere is perfectly transmissive at solar wavelengths and perfectly absorptive at IR wavelengths
• The atmosphere emits as a perfect blackbody at IR wavelengths
• The atmosphere and surface can have different temperatures \(T_a\) and \(T_s\)

Top Radiative Balance

• No IR radiation emitted by the surface is transmitted to space because all is absorbed by the atmosphere
• Downward radiation flux is \(S_0 / 4\)
• One component of upward flux is reflected solar radiation: \(\alpha_p S_0 / 4\)
• The other component of upward flux is IR radiation emitted by the atmosphere: \(\sigma T_a^4\)
Top Radiative Balance

For radiative balance:
\[ \frac{S_0}{4} = \alpha_p \frac{S_0}{4} + \sigma T_s^4 \]
This can be rearranged as:
\[ (1 - \alpha_p) \frac{S_0}{4} = \sigma T_s^4 \]
Note that \( T_s \) is the same as \( T_e \).

Atmosphere Radiative Balance

- The atmosphere absorbs no solar radiation
- The atmosphere absorbs all IR radiation emitted by the surface: \( \sigma T_s^4 \)
- The atmosphere emits IR radiation in both the upward and downward directions:
  \[ \sigma T_s^4 + \sigma T_a^4 \]

Atmosphere Radiative Balance

For radiative balance:
\[ \sigma T_s^4 = \sigma T_s^4 \text{ (up)} + \sigma T_a^4 \text{ (down)} \]
This can be rearranged as:
\[ \sigma T_s^4 = 2 \sigma T_a^4 \]

Surface Radiative Balance

- The surface absorbs some solar radiation:
  \( (1 - \alpha_p) \frac{S_0}{4} \)
- The surface absorbs all IR radiation that the atmosphere emits downward: \( \sigma T_a^4 \)
- The surface emits radiation: \( \sigma T_s^4 \)

Surface Radiative Balance

For radiative balance:
\[ (1 - \alpha_p) \frac{S_0}{4} + \sigma T_s^4 = \sigma T_s^4 \]
This can be rearranged as:
\[ (1 - \alpha_p) \frac{S_0}{4} = \sigma T_s^4 - \sigma T_a^4 \]

The Simple Atmosphere

We now have a simple model of Earth’s climate that includes an atmosphere:
\[ (1 - \alpha_p) \frac{S_0}{4} = \sigma T_s^4 \]
\[ \sigma T_s^4 = 2 \sigma T_a^4 \]
\[ (1 - \alpha_p) \frac{S_0}{4} = \sigma T_s^4 - \sigma T_a^4 \]
What are values of $T_a$ and $T_s$?

Obtain one equation such that $T_a$ is on the left side, $T_s$ is eliminated, and everything else is on the right side

Obtain a second equation such that $T_s$ is on the left side, $S_0$ and $\alpha_p$ are eliminated, and everything else is on the right side

\[
T_a = \left[ \frac{(1 - \alpha_p) S_0}{4\sigma} \right]^{1/4}
\]

\[
T_s = (2)^{1/4} T_a
\]

What are values of $T_a$ and $T_s$?

A Simple IR Window

A Simple IR Window

- Note that $T_s$ is greater than $T_a$ and $T_o$
- The atmosphere keeps the surface warmer than it would be if no atmosphere were present
- This is a mathematical model of the greenhouse effect

- Now allow the atmosphere to transmit to space some of the IR radiation emitted by the surface
- Since the atmosphere is no longer perfectly absorptive at IR wavelengths, it also no longer emits as a perfect blackbody at IR wavelengths

Emissivity

- The parameter $\varepsilon$ is called the emissivity of the atmosphere (the fraction of radiation emitted relative to a blackbody)
- The absorptivity of the atmosphere also has the value of $\varepsilon$ (the fraction of incident radiation that is absorbed)
A Simple IR Window

Top Radiative Balance

• Downward radiation flux is \( S_0 / 4 \)
• One component of upward flux is reflected solar radiation: \( \alpha_p \ S_0 / 4 \)
• Another component is IR radiation emitted by the atmosphere: \( \varepsilon \ \sigma \ T_a^4 \)
• A third component is IR radiation emitted by the surface and not absorbed by the atmosphere: \( (1 - \varepsilon) \ \sigma \ T_s^4 \)

Top Radiative Balance

For radiative balance:
\[
\frac{S_0}{4} = \alpha_p \frac{S_0}{4} + \varepsilon \ \sigma \ T_a^4 + (1 - \varepsilon) \ \sigma \ T_s^4
\]
This can be rearranged as:
\[
(1 - \alpha_p) \frac{S_0}{4} = \varepsilon \ \sigma \ T_a^4 + (1 - \varepsilon) \ \sigma \ T_s^4
\]
Note that \( T_a \) is no longer the same as \( T_e \)

Atmosphere Radiative Balance

For radiative balance:
\[
\varepsilon \ \sigma \ T_s^4 = \varepsilon \ \sigma \ T_a^4 \ (\text{up}) + \varepsilon \ \sigma \ T_a^4 \ (\text{down})
\]
This can be rearranged as:
\[
\varepsilon \ \sigma \ T_s^4 = 2 \varepsilon \ \sigma \ T_a^4
\]

Surface Radiative Balance

• The surface absorbs some solar radiation: \( (1 - \alpha_p) \ S_0 / 4 \)
• The surface absorbs all IR radiation that the atmosphere emits downward: \( \varepsilon \ \sigma \ T_a^4 \)
• The surface emits radiation: \( \sigma \ T_s^4 \)
Surface Radiative Balance

For radiative balance:
\[(1 - \alpha_p) S_0 / 4 + \varepsilon \sigma T_a^4 = \sigma T_s^4\]
This can be rearranged as:
\[(1 - \alpha_p) S_0 / 4 = \sigma T_s^4 - \varepsilon \sigma T_a^4\]

The Simple IR Window

We now have a simple model of Earth’s climate that includes an atmosphere that is partially transmissive at IR wavelengths:
\[(1 - \alpha_p) S_0 / 4 = \varepsilon \sigma T_a^4 + (1 - \varepsilon) \sigma T_s^4\]
\[\varepsilon \sigma T_s^4 = 2 \varepsilon \sigma T_a^4\]
\[(1 - \alpha_p) S_0 / 4 = \sigma T_s^4 - \varepsilon \sigma T_a^4\]

How are \(T_s\) and \(\varepsilon\) related?

Obtain one equation such that \(T_s\) is on the left side, \(T_a\) is eliminated, and everything else is on the right side

\[T_s = \left[\frac{(1 - \alpha_p) S_0}{(4 - 2\varepsilon) \sigma}\right]^{1/4}\]

How are \(T_s\) and \(\varepsilon\) related?

Obtain one equation such that \(T_s\) is on the left side, \(T_a\) is eliminated, and everything else is on the right side

How are \(T_s\) and \(\varepsilon\) related?

• Note that \(\varepsilon = 0\) corresponds to no effective atmosphere and \(\varepsilon = 1\) corresponds to a perfectly absorbing atmosphere
• For \(0 < \varepsilon < 1\), \(T_s\) is warmer than \(T_a\) and colder than \(T_s\) for the perfectly absorbing atmosphere
• Changes in \(\varepsilon\) represent changes in greenhouse gas concentrations

How are \(T_s\) and \(\varepsilon\) related?

• What value of \(\varepsilon\) will produce a value of \(T_s\) equal to current global surface temperature (about 288 K or 15°C)?
• How close is this to the real fraction of radiation absorbed by the atmosphere?
• How much does \(\varepsilon\) need to change to produce a 1 K increase in \(T_s\)?
• What about 2 K? 5 K? A decrease in \(T_s\)?
Non-Equilibrium

- The climate does not have an instantaneous response to a change in emissivity
- What is the transient behavior of the atmosphere before it comes to equilibrium?

A Simple Climate Model

- Let the Earth be covered by a “swamp” ocean with uniform depth $h$, density $\rho$, and specific heat $c$
  \[
  \text{thermal inertia of ocean} = \rho \ c \ h
  \]
- Let $F$ be the net radiation flux at the Earth’s surface
  \[
  F = (1 - \alpha_p) \ S_0 / 4 + \varepsilon \ \sigma \ T_s^4 - \sigma \ T_s^4
  \]

A Simple Climate Model

- Assume $\varepsilon$ is known as a function of time
- If $T_s$ is known at time $t_0$, it is simple to calculate $T_s$ at time $t_0 + \Delta t$
- The value of $T_s$ at time $t_0 + \Delta t$ can then be used to calculate $T_s$ at time $t_0 + 2\Delta t$
- Etc.

A Simple Climate Scenario

- Let the climate initially be in equilibrium
- Let $\varepsilon$ change instantaneously from 0.8 to 0.85 and thereafter remain constant
- Let $h$ be 4000 m (approximate average ocean depth)
- Let $\rho$ be 1025 kg m$^{-3}$ and $c$ be 3850 J kg$^{-1}$ K$^{-1}$ (typical values for seawater)